The negative sign in Eq. (4-45) indicates that water level in an observation well falls when barometric pressure increases, and vice versa.

For an unconfined aquifer, compressibility of aquifer material and water is relatively less significant compared to changes in water volume that result from water table fluctuations. The changes in atmospheric pressure are transmitted directly and simultaneously to the water table and observation well. Therefore, there is little or no effect of barometric pressure fluctuations on water levels observed in the well. The change in water level in the well is almost the same as in the unconfined aquifer. It has been observed that fluctuations in barometric pressure may result in small fluctuations in the water table in unconfined aquifers.

Example 4-10: Estimate the change in water level in a well fully penetrating a confined aquifer when barometric pressure changes by 7.72 cm of mercury. Assume $\alpha_s = 11.8 \times 10^{-6}$ cm^2/kg; $\phi = 0.35$; and $\beta = 47 \times 10^{-6}$ cm^2/kg.

Solution: $(dp_a/\gamma) = 0.0772 \times 13.6 = 1.05$ m of water. Using Eq. (4-45), $dh/(dp_a/\gamma) = dh/(1.05) = -1/[1 + \{11.8/(0.35 \times 47)\}] = -0.582$. The negative sign indicates that increase in barometric pressure results in depressing the water level in the well. Thus, $dh = -1.05 \times 0.582 = -0.61$ m.

Subsidence

The stress caused by total weight of soil and water above a point in an aquifer is balanced by effective (compressive) stress on the aquifer material and fluid (hydrostatic) pressure (Delleur 1999):

$$p_T = \sigma + p_w \tag{4-46}$$

where

p_T = total pressure due to weight of soil and water

p_w = fluid (hydrostatic) pressure

σ = effective or compressive stress on aquifer material

The hydrostatic pressure can be measured by a piezometer. An increase in the compressive stress on the aquifer material causes reduction in its volume (or in its thickness in one-dimensional compression), which may result in subsidence. Excessive groundwater extraction may result in some reduction in the total pressure, but it could result in relatively greater reduction in the hydrostatic pressure in the aquifer. This may result in an increase in the compressive stress on the aquifer material and cause land subsidence. Groundwater pumping has to be controlled to minimize potential for subsidence. Computational steps for preliminary estimates of subsidence due to lowering of groundwater levels in an aquifer are listed below.

1. Estimate total load (pressure) at the position of lowered groundwater level before pumping due to the weight of overlying soils and water held in pores.

2. Estimate hydrostatic pressure at that level due to head of groundwater above that level.
3. Estimate compressive stress on aquifer material at that level as the difference of steps (1) and (2) before pumping.
4. Following the same procedure, estimate compressive stress at the same level after lowering of the groundwater level.
5. Find the difference, $\Delta\sigma$, in the initial and final compressive stresses at that level [(4) − (3)].
6. The change in compressive stress at the level of the initial groundwater level is zero. Thus, the average change in compressive stress in the aquifer column between these two levels is $\Delta\sigma/2$.
7. Estimate subsidence, δ, in this aquifer column between the initial and lowered groundwater levels,

$$\delta = \alpha_s(\Delta\sigma/2)\Delta h \tag{4-47a}$$

where

α_s = compressibility of aquifer material

Δh = change in groundwater levels

8. Change in compressive stress in aquifer material below the lowered groundwater level will be $\Delta\sigma$.
9. If there are two or more layers of soils below the lowered groundwater level, estimate subsidence in each:

$$\delta_1 = \alpha_1(\Delta\sigma)L_1, \quad \text{and} \quad \delta_2 = \alpha_2(\Delta\sigma)L_2, \text{ etc.} \tag{4-47b}$$

where

δ_1, δ_2 = subsidence in layers 1 and 2

α_1, α_2 = compressibility of layers 1 and 2

L_1, L_2 = thickness of layers 1 and 2, respectively

10. Estimate total subsidence at the bottom of layer 2 = $\delta + \delta_1 + \delta_2$.

Example 4-11: Extensive groundwater pumping in an area is expected to lower groundwater levels by 25 m. Initial water level is 10 m below the ground surface. The aquifer material is sand up to a depth of 50 m below the ground surface. Below the sand is a 30-m-thick silty clay layer overlying the bedrock. Estimate potential subsidence in the soils above the bedrock. Assume degree of saturation in the unsaturated soil zone above groundwater level to be 0.10, unit weight of sand grains = 2,600 kg/m^3; unit weight of water = 1,000 kg/m^3; porosity of sand = 0.33; compressibility of sand = 12×10^{-8} m^2/kg; and compressibility of silty clay = 100×10^{-8} m^2/kg.

Solution: Let the positions of initial and lowered groundwater level be denoted by B and C, the bottom of the sand unit by D, and the bottom of clay by E.

Vertical distance between ground surface and C = 10 + 25 = 35 m.

After expected lowering of groundwater level, total load above point C at the elevation of lowered groundwater level = 35 × (1 − 0.33) × 2,600 + 35 × (0.33 × 0.10) × 1,000 = 60,970 + 1,155 = 62,125 kg/m^2.

Hydrostatic pressure at point C (after lowering of the water table to this level) = 0.

Compressive stress in soils at this elevation (at point C) = 62,125 − 0 = 62,125 kg/m^2.

Under initial groundwater conditions, total load above point C = 10 × (1 − 0.33) × 2,600 + 10 (0.33 × 0.10) × 1,000 + 25 × (1 − 0.33) × 2,600 + 25 × 0.33 × 1,000 = 17,420 + 330 + 43,550 + 8,250 = 69,550 kg/m^2.

Under initial groundwater conditions, hydrostatic pressure at C = 25 × 1,000 = 25,000 kg/m^2.

Under initial groundwater conditions, compressive stress in soils at C = 69,550 − 25,000 = 44,550 kg/m^2.

Increase in compressive stress at C due to lowering of water level = 62,125 − 44,550 = 17,575 kg/m^2.

Change in compressive stress in soils at initial groundwater level (point B) = 0.

Change in compressive stress at lowered groundwater level (point C) = 17,575 kg/m^2.

Average subsidence over a soil column of Δh = 25 m is given by $\delta = \alpha_s(\Delta\sigma/2)\Delta h = 12 \times 10^{-8} \times (17{,}575/2) \times 25 = 0.026$ m.

For sand below the lowered groundwater level (point C to D), $\alpha_1 = 12 \times 10^{-8}$ m^2/kg and L_1 = 15 m. So, $\delta_1 = \alpha_1(\Delta\sigma)L_1 = 12 \times 10^{-8} \times 17{,}575 \times 15 = 0.032$ m.

For silty clay below the sand (point D to E), $\alpha_2 = 100 \times 10^{-8}$ m^2/kg, and L_2 = 30 m. So, $\delta_1 = \alpha_1(\Delta\sigma)L_1 = 100 \times 10^{-8} \times 17{,}575 \times 30 = 0.527$ m.

Total subsidence above bedrock = 0.026 + 0.032 + 0.527 = 0.585 m.

Safe Yield, Specific Capacity, and Efficiency

There is no precise definition of the safe yield of an aquifer or well. Usually, it is the rate of groundwater withdrawal that can be maintained without creating potential for subsidence; saltwater intrusion; undue depletion of the groundwater table, which cannot be replenished by natural groundwater recharge; undue interference with the yield of existing groundwater wells; undue induced recharge from nearby surface water bodies; and encroachment on existing groundwater contaminant plumes. The limitations on drawdowns at specific locations may be specified by local, state, or federal regulatory agencies. Sometimes, the term "optimum yield" is used to identify safe yield, which can be economically obtained if various alternative groundwater resource management strategies are considered.

Specific capacity of a well is its yield per unit drawdown (Q/s), where s includes drawdown in the aquifer at the boundary of the well screen and well loss resulting from turbulent flow of groundwater through the well screen. The efficiency of a well is the ratio of its actual specific capacity to the theoretical specific capacity. It is also defined as the ratio of the aquifer head loss to the total head loss. The total head loss (or total drawdown at the well face) includes aquifer (i.e., theoretical) drawdown and well drawdown. Aquifer drawdown varies linearly with discharge and can be estimated by steady-state or non–steady-state equations for drawdown at the well face (e.g., Eqs. (4-17) and (4-81)). Well loss includes a linear component and a nonlinear component. The linear component includes drawdown in the gravel pack and screen entrance, and the nonlinear component includes losses due to turbulent flow in the well. A simple method to estimate well efficiency is as follows:

- Plot drawdown, s, on the y-axis on the natural scale, and plot the distance from the well, r, on the logarithmic scale on the x-axis.
- Draw the best-fitting straight line through these points by visual judgment.
- Extend the straight line to $r = r_w$ (well radius) and read the theoretical drawdown, s_0, at this location.
- Estimate well efficiency = s_0/s_w, where s_w is the actual drawdown measured in the well.

In the case of an unconfined aquifer, well operation may cause considerable reduction in the saturated thickness of the aquifer. As a result, extra drawdown is observed. This extra drawdown does not represent inefficiency in the well. If the reduction in saturated thickness is more than 20%, then the drawdown, s_0, may be corrected before well efficiency is computed (see the section in this chapter entitled "Unsteady Radial Flow to a Well Fully Penetrating an Unconfined Aquifer"):

$$s_0 \text{ (corrected)} = s_0 - \{s_0^2/(2H)\} \tag{4-48}$$

where H = initial saturated thickness of the aquifer. The corrected drawdown should be used in preparing the aforementioned plot. A well efficiency of about 70 to 80% is acceptable for a well-designed well.

Transient (Unsteady) Groundwater Flow

Unsteady One-Dimensional Flow

Continuity equation for unsteady one-dimensional groundwater flow in a confined aquifer of thickness, B, and hydraulic conductivity, K, is

$$\partial^2 h/\partial x^2 = S/T\, \partial h/\partial t \tag{4-49}$$

where

S = dimensionless storage coefficient or storativity

T = transmissivity or transmissibility of the aquifer

$S_s = S/B$ = specific storage, defined as the volume of water that is released by a unit volume of the aquifer per unit decline in hydraulic head:

$$S = \rho\, g(\alpha_s + \theta\, \beta)B \tag{4-50}$$

$$T = K\, B \tag{4-51}$$

Typical values of specific storage, S_s, are given in Table 4-8 (USEPA 1985).

Table 4-8. Typical values of specific storage

Material	Specific storage (cm^{-1})
Plastic clay	$2.0 \times 10^{-4} - 2.5 \times 10^{-5}$
Stiff clay	$2.5 \times 10^{-5} - 1.3 \times 10^{-5}$
Medium hard clay	$1.3 \times 10^{-5} - 6.9 \times 10^{-6}$
Loose sand	$9.8 \times 10^{-6} - 5.1 \times 10^{-6}$
Dense sand	$2.1 \times 10^{-6} - 1.3 \times 10^{-6}$
Dense sandy gravel	$9.8 \times 10^{-7} - 5.1 \times 10^{-7}$
Rock (fissured, jointed)	$6.9 \times 10^{-7} - 3.2 \times 10^{-8}$
Sound rock	Less than 3.2×10^{-8}

Source: USEPA (1985).

Linearized continuity equation for non–steady-state one-dimensional flow in an unconfined aquifer is

$$\partial^2 h/\partial x^2 = [S_y/(KH)]\, \partial h/\partial t \qquad (4\text{-}52)$$

where

S_y = specific yield

H = average saturated thickness

Specific yield is that portion of water held in soil pores that can be extracted from the aquifer per unit area per unit drop in the water table. It is also called effective porosity and is less than total porosity. Typical values of specific yield are given in Table 4-9 (USEPA 1985).

Bank Storage

During flood stages, river water enters the porous material of the banks. As a result, the groundwater table on both sides of the river may rise. After the flood recedes, water surface elevation in the river returns to the normal condition in a relatively short time. The water stored in the bank material is slowly released into the stream under the head difference between the elevated water table and the normal (lowered) water surface elevation in the stream. In some cases, bank material may consist of contaminated sediments resulting from past industrial activities, and the water returning to the river by way of seepage from banks also may be contaminated. In certain situations, estimation of the rates of return flow and quantities of contaminated groundwater likely to enter the river are required. Analytical equations for preliminary analysis of this type of situations are given below (Carslaw and Jaeger 1984; Glover 1985):

1. $h = H\,\mathrm{erf}[x/\sqrt{(4\alpha t)}]$ (4-53)
2. $q(x, t) = \{HKD/\sqrt{(\pi\alpha t)}\} \exp\{-x^2/(4\alpha t)\}$ (4-54)
3. $q(0, t) = \{HKD/\sqrt{(\pi\alpha t)}\}$ (4-55)
4. $Q(t) = 2HKD\sqrt{\{t/(\pi\alpha)\}}$ (4-56)

where

H = height of the water table resulting from the flood stage, measured above normal (lowered) water surface elevation in the river

h = height of the water table above normal (lowered) water surface elevation in the river at distance, x, and time, t

x = distance into the bank from the edge of the river

t = time after the flood event (i.e., after the river has receded to normal water surface elevation)

α = aquifer diffusivity = KD/S_y

D = initial saturated thickness of the bank material (i.e., depth of raised water surface in the bank above the impervious base)

$q(x, t)$ = rate of return flow at distance, x, and time, t, per unit length of bank (parallel to river flow)

$q(0, t)$ = rate of return flow at the edge of the river at time, t, per unit length of bank

$Q(t)$ = total volume of flow that has returned to the river up to time, t, per unit length of the bank

Table 4-9. Typical values of specific yield

Material	Range (%)	Mean (%)
Clay	1.1–17.6	6
Silt	1.1–38.6	20
Fine sand	1.0–45.9	33
Medium sand	16.2–46.2	32
Coarse sand	18.4–42.9	30
Fine gravel	12.6–39.9	28
Medium gravel	16.9–43.5	24
Coarse gravel	13.2–25.2	21
Loess	14.1–22.0	18
Dune sand	32.3–46.7	38
Till (predominantly silt)	0.5–13.0	6
Till (predominantly sand)	1.9–31.2	16
Till (predominantly gravel)	5.1–34.2	16
Glacial drift (predominantly silt)	33.2–48.1	40
Glacial drift (predominantly sand)	29.0–48.2	41
Sand stone (fine-grained)	2.1–39.6	21
Sandstone (medium-grained)	11.9–41.1	27
Siltstone	0.9–32.7	12
Shale	0.5–5.0	—
Limestone	0.2–35.8	14
Schist	21.9–33.2	26

Source: USEPA (1985).

The notation erf[$x/\sqrt{(4\alpha t)}$] represents the error function of the quantity within the brackets. The values of error function are available in tabulated form (e.g., Abramowitz and Stegun 1972). A rational approximation suitable for computer use is

$$\text{erf}(x) = 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5)\exp(-x^2) \tag{4-57}$$

where

$$t = 1/(1 + 0.3275911x)$$
$$a_1 = 0.254829592$$
$$a_2 = -0.284496736$$
$$a_3 = 1.421413741$$
$$a_4 = -1.453152027$$
$$a_5 = 1.061405429$$

Eqs. (4-53) to (4-56) are based on the assumption that bank material is isotropic and homogeneous and the river returns to normal water surface elevation in a relatively short time following the flood event so that water table in the bank material is still at the elevated level.

Example 4-12: During flood season, the water table in the banks of a stream is found to have risen by 3 m above the normal water surface elevation of the river. Estimate return flow from one side of the bank after the river has returned to normal water surface elevation, which may be assumed to have occurred in a relatively short time. The bank material on one side of the stream contains 0.4 gm/kg of adsorbed lead. The distribution coefficient of lead is found to be 10,000 l/kg. Estimate the mass of lead that may have entered the river by leaching from the bank materials in a period of 90 days. Assume that adsorbed lead is in a soluble state and can be leached during this period. For the bank materials, use $KD = 1{,}766$ m^2/day and $S_y = 0.15$.

Solution: $\alpha = KD/S_y = 1{,}766/0.15 = 11{,}773.3$ m^2/day.

From Eq. (4-55), $q(0, t) = 3 \times 1{,}766/\sqrt{(\pi \times 11{,}773.3 \times 90)} = 2.9$ m^3/day per meter length of bank.

From Eq. (4-56), $Q(t) = 2 \times 3 \times 1{,}766\sqrt{\{(90/(\pi \times 11{,}773.3)\}} = 522.7$ m^3 per meter length of bank.

Assuming equilibrium conditions, $S_d = K_d C$, where S_d = mass of lead adsorbed per unit dry mass of bank material = 0.0004 kg/kg; K_d = distribution coefficient for lead = 10 m^3/kg; and C = concentration of lead in water contained in bank materials. So, $C = 0.0004/10 = 0.00004$ kg/m^3.

Mass of lead entering the river in 90 days = 0.00004 × 522.7 = 0.02 kg per meter length of bank.

This estimate is preliminary because establishment of equilibrium conditions may take longer and leaching of lead may be slower.

Flow toward Drains and Drain Spacing

An approximate method for spacing of tile drains in agricultural areas was given in a previous section entitled "Darcian Flow." If an initial elevated water table is to be lowered to a certain level within a specified period, then drain spacing has to be estimated using the transient flow equation (Eq. (4-49)). Relevant analytical equations for this case are listed below (Carslaw and Jaeger 1984; Glover 1985):

1. $h(x, t) = (4H/\pi)\,\Sigma\{1/(2n+1)\}\,[\exp\{-(2n+1)^2\pi^2\alpha\, t/L^2\}]\,\sin\{(2n+1)\,\pi x/L\},\ n = 0, 1, 2, \ldots, \propto$ (4-58)
2. $h(x = L/2) = (4H/\pi)\,\Sigma\{1/(2n+1)\}\,[\exp\{-(2n+1)^2\pi^2\alpha\, t/L^2\}]\,\sin\{(2n+1)\,\pi/2\},\ n = 0, 1, 2, \ldots, \propto$ (4-59)
3. $q(x = 0, t) = (4\,KDH/L)\,\Sigma[\exp\{-(2n+1)^2\pi^2\alpha\, t/L^2\}],\ n = 0, 1, 2, \ldots, \propto$ (4-60)
4. $p = (8/\pi^2)\,\Sigma[\exp\{-(2n+1)^2\pi^2\alpha\, t/L^2\}]/(2n+1)^2,\ n = 0, 1, 2, \ldots, \propto$ (4-61)

where

H = height of initial water table above drains

D = height of drains above impervious layer

L = drain spacing

$h(x, t)$ = height of water table at distance, x, and time, t, above drains

x = distance from drain

t = time since groundwater starts to drain from initial water table elevation

$q(x = 0, t)$ = flow to drain from one side per unit length of drain

p = fraction of drainable volume of water that remains to be drained at time, t

It is assumed that $H \ll D$. Otherwise, D may be taken to be the average saturated thickness of the aquifer. The minimum lowering of the water table will occur at the center of two parallel drains, i.e., at $x = L/2$. Thus, Eq. (4-59) can be used to estimate drain spacing for a minimum water table lowering to h at $x = L/2$ above the drains. The infinite series of Eqs. (4-58) to (4-61) converge fairly rapidly for $(\alpha t/L^{2)} \gg 0.01$. For such cases, the second term is <2% of the first, and the remaining terms are even smaller. Thus, these equations may be approximated by

1. $h(x, t) = (4H/\pi)[\exp\{-\pi^2\alpha\, t/L^2\}]\,\sin\{\pi\, x/L\}$ (4-62)
2. $h(x = L/2) = (4H/\pi)[\exp\{-\pi^2\alpha\, t/L^2\}]$ (4-63)
3. $q(x = 0, t) = (4\,K\,D\,H/L)[\exp\{-\pi^2\alpha\, t/L^2\}]$ (4-64)
4. $p = (8/\pi^2)[\exp\{-\pi^2\alpha\, t/L^2\}]$ (4-65)

From Eq. (4-63), for $(\alpha\, t/L^{2)} \gg 0.01$, $L = \pi\sqrt{[\alpha\, t/\ln\{4\,H/(\pi h)\}]}$ (4-66)

For other cases where $(\alpha t/L^2) \leq 0.01$, $n = 0, 1, 2$, and 3 may have to be used in Eqs. (4-58) to (4-61). Usually, terms involving $n > 3$ may be too small to consider.

Example 4-13: In an irrigated area, the impervious soil layer is about 12 m below the field level. Tile drains are to be installed about 3 m below the field level. During the first irrigation season, the water table rises to within 0.75 m below the field level. The next irrigation period is 30 days after the first. Before the second irrigation, the water table has to be lowered to a minimum of 1.5 m below the field level. Estimate drain spacing for this situation. Use K = 3.05 m/day and S_y = 0.18.

Solution: Height of maximum water table above drains = $H = 3.0 - 0.75 = 2.25$ m.

Height of maximum water table above impervious layer = $12 - 0.75 = 11.25$ m.

Height of drains above impervious layer = $12 - 3 = 9$ m, and $h(x = L/2) = 3 - 1.5 = 1.5$ m.

Average saturated thickness = $D \cong (11.25 + 9)/2 = 10.125$ m, and $\alpha = KD/S_y = 3.05 \times 10.125/0.18 = 171.56$ m^2/day.

Using Eq. (4-66), $L = \pi\sqrt{[(171.56 \times 30)/ \ln \{(4 \times 2.25)/(\pi \times 1.5)\}]} = 280.2$ m.

Check the validity of Eq. (4-66), $\alpha t/L^2 = 171.56 \times 30/(280.2)^2 = 0.066$. So, the approximation of Eq. (4-66) is valid.

The following is an approximate equation to estimate steady-state groundwater flow toward a single circular drain or tunnel (Freeze and Cherry 1979):

$$q_T = 2\pi K H/\ln(2H/r) \tag{4-67}$$

where

q_T = flow into the drain or tunnel per unit length

H = head above tunnel centerline

r = radius of drain or tunnel

An approximate equation for the transient case is as follows (Freeze and Cherry 1979):

$$q_T(t) = \sqrt{(C K H^3 S_y t)} \tag{4-68}$$

where

$q_T(t)$ = flow into the drain or tunnel per unit length at time, t, after the breakdown of steady flow

C is a constant

The values of C may vary from 4/3 to 2. Eqs. (4-67) and (4-68) may be useful for preliminary analyses. Numerical models must be used for more refined analyses.

At some industrial sites, trenches or underground drains are provided to collect or intercept contaminated groundwater from the site area, which may be pumped out through sumps located at suitable locations on the trench or drain. The pumps and trenches are designed for groundwater flows that may be expected during high groundwater table conditions following storm events. If, initially, the water table is approximately horizontal, the following equations estimate the lowering of groundwater levels and flows entering the trench or drain (Carslaw and Jaeger 1984):

1. $h\ (x, t) = h_0 + [4(H - h_0)/\pi]\ \Sigma\{(-1)^n/(2\ n + 1)\}\ [\exp\{-(2n + 1)^2\ \pi^2\ \alpha t/(4\ L^2)\}]\ \cos\{(2n + 1)\ \pi\ x/(2L)\},\ n = 0, 1, 2, \ldots, \propto$ (4-69)
2. $q\ (x = 0, t) = \{2\ KD(H - h_0)\}/L\Sigma\ [\exp\{-(2n + 1)^2\ \pi^2\ \alpha\ t/(4\ L^2)\}],$ $\mathrm{n} = 0, 1, 2, \ldots, \propto$ (4-70)
3. $Q\ (t) = \{8\ K\ D\ L\ (H - h_0)\}/(\pi^2\ \alpha)\ \Sigma\ [\exp\{-(2n + 1)^2\ \pi^2\ \alpha\ t/(4\ L^2)\}]/(2n + 1)^2,\ n = 0, 1, 2, \ldots, \propto$ (4-71)

As in the case of Eqs. (4-55) to (4-58), for $\alpha\ t/(4\ L^2) > 0.01$, simplified approximate equations are as follows:

1. $h\ (x, t) = h_0 + [4\ (H - h_0)/\pi)][\exp\{-\pi^2\ \alpha\ t/(4\ L^2)\}]\ \cos\{\pi\ x/(2\ L)\}$ (4-72)
2. $q\ (x = 0, t) = [\{2\ K\ D\ (H - h_0)\}/L]\ [\exp\{-\pi^2\ \alpha\ t/(4\ L^2)\}]$ (4-73)
3. $Q\ (t) = [\{8\ K\ D\ L\ (H - h_0)\}/(\pi^2\ \alpha)]\ [\exp\{-\pi^2\ \alpha\ t/(4\ L^2)\}]$ (4-74)

where

H = height of initial water table above impervious layer

D = average saturated thickness

L = distance from groundwater divide to trench or drain

$h(x, t)$ = height of water table above impervious layer at distance, x, and time, t

x = distance from groundwater divide

t = time since groundwater starts to drain from initial water table elevation

h_0 = height of drain above impervious layer

$q\ (x = 0, t)$ = flow to trench from one side per unit length of trench

$Q(t)$ = total flow that entered the trench from one side up to time, t

If initial water table can be approximated by a sloping straight line, then groundwater lowering due to the trench can be estimated by

$$h\ (x, t) = h_0 + [8(H - h_0)/\pi^2]\ \Sigma\{1/(2n + 1)^2\}\ [\exp\{-(2n + 1)^2\ \pi^2\ \alpha\ t/(4\ L^2)\}]\ \cos\{(2n + 1)\ \pi\ x/(2\ L)\},\ n = 0, 1, 2, \ldots, \propto \qquad (4\text{-}75)$$

or

$$h\ (x, t) = h_0 + (H - h_0)\{(L - x)/L\} - [2(H - h_0)\ \surd(\alpha\ t)/L]\ \Sigma(-1)^n\ \{\mathrm{ierfc}\ (U_1) - \mathrm{ierfc}\ (U_2)\},\ n = 0, 1, 2, \ldots, \propto \qquad (4\text{-}76)$$

where

$U_1 = (2nL + x)/\surd(4\alpha t)$

$U_2 = \{(2n + 2)L - x\}/\surd(4\alpha t)$