

$$L_{el} = \sqrt[4]{\frac{E_1 \times h^3}{12 \times (1 - \mu^2) \times k}} \quad (B.15)$$

where

- $E_1$  is the Young's modulus [N/mm<sup>2</sup>] of slab/pavement – 1st layer;
- $h$  is the thickness of the slab/pavement [mm] or  $h_I$  or  $h_{II}$  or  $h_{III}$ ;
- $\mu$  is the Poisson's ratio of the slab/pavement material;
- $k$  is the bedding modulus [N/mm<sup>3</sup>].

Longitudinal and lateral bending moments ( $M_{long}$  and  $M_{lat}$ ) activated by adjacent rail seat loads  $P_j$  at distance  $x_j$  from reference load  $P_0$  may be computed from the radial and tangential moments  $M_{r,t}$ . Radial and tangential moments activated by neighbouring rail seat loads are given by:

$$M_{j-r,t} = \lambda_{r,t} \times P_j \quad (B.16)$$

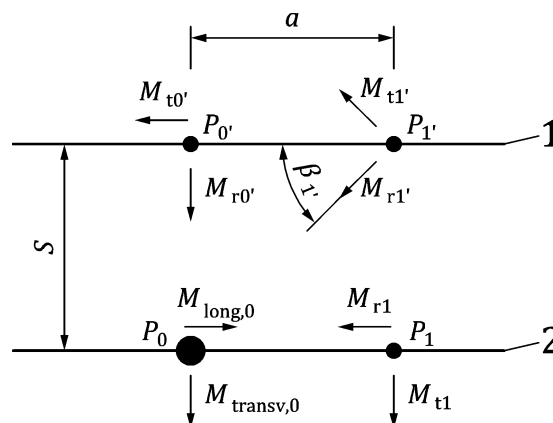
The rail seat loads  $P_j$  [N] are calculated according to Annex A and to compute  $\lambda_{r,t}$  Formulae (B.17) and (B.18) shall be used:

$$\lambda_{r,j} = 0,160 - 0,284 \times \left(\frac{x_j}{L_{el}}\right) + 0,157 \times \left(\frac{x_j}{L_{el}}\right)^2 - 0,036 \times \left(\frac{x_j}{L_{el}}\right)^3 + 0,003 \times \left(\frac{x_j}{L_{el}}\right)^4 \quad (B.17)$$

$$\lambda_{t,j} = 0,2499 - 0,3511 \times \left(\frac{x_j}{L_{el}}\right) + 0,2034 \times \left(\frac{x_j}{L_{el}}\right)^2 - 0,0536 \times \left(\frac{x_j}{L_{el}}\right)^3 + 0,0052 \times \left(\frac{x_j}{L_{el}}\right)^4 \quad (B.18)$$

As an example, the above computation for the Figure B.9 of the influence of neighboring loads  $P_1$ ,  $P_1'$ , and  $P_0'$  on  $P_0$ , are shown in Figure B.10 and Formulae (B.17) and (B.18).

For each rail seat location of interest, the radial and tangential bending moments ( $M_r$  and  $M_t$ ) shall be computed.



Key

- 1 left rail
- 2 right rail

**Figure B.10 — Example of configuration of radial and tangential bending moments**

Right rail:

$$P1: M_{\text{long},1} = \lambda_{r,1} \times P_1 \quad (\text{B.19})$$

$$M_{\text{lat},1} = \lambda_{t,1} \times P_1 \quad (\text{B.20})$$

Left rail:

$$P0: M_{\text{long},0'} = \lambda_{t,0'} \times P_{0'} \quad (\text{B.21})$$

$$M_{\text{lat},0'} = \lambda_{r,0'} \times P_{0'} \quad (\text{B.22})$$

$$P1: M_{\text{long},1'} = \lambda_{r,1'} \times P_{1'} + 0,5 \times (\lambda_{t,1'} \times P_{1'} - \lambda_{r,1'} \times P_{1'}) \times [1 - \cos(2 \times \beta_{1'})] \quad (\text{B.23})$$

$$M_{\text{lat},1'} = \lambda_{r,1'} \times P_{1'} + 0,5 \times (\lambda_{t,1'} \times P_{1'} - \lambda_{r,1'} \times P_{1'}) \times [1 + \cos(2 \times \beta_{1'})] \quad (\text{B.24})$$

$$P0: M_{\text{long, neigh}} = \Sigma M_{\text{long},j} \quad (\text{B.25})$$

$$M_{\text{lat, neigh}} = \Sigma M_{\text{lat},j} \quad (\text{B.26})$$

For the above formulae:

$\lambda_{r,1}$  and  $\lambda_{t,1}$  are defined using  $x_1 = a$ ;

$\lambda_{r,0'}$  and  $\lambda_{t,0'}$  are defined using  $x_{0'} = s$ ;

$\lambda_{r,1'}$  and  $\lambda_{t,1'}$  are defined using  $x_{1'} = \frac{a}{\cos \beta_{1'}}$  and  $\beta_{1'} = \tan^{-1}(\frac{s}{a})$ .

### B.1.5.3 Longitudinal bending tensile stress $\sigma_{\text{long}}$ [N/mm<sup>2</sup>] and lateral bending tensile stress $\sigma_{\text{lat}}$ [N/mm<sup>2</sup>] due to rail seat loads

#### System variant I: single layer (thickness $h_I$ ) on substructure

Longitudinal and lateral bending tensile stress at bottom of slab:

$$\sigma_{\text{long I}} = \frac{6 \times M_{\text{long I}}}{h_1^2} \text{ and } \sigma_{\text{lat I}} = \frac{6 \times M_{\text{lat I}}}{h_1^2} \quad (\text{B.27})$$

#### System variant II: unbonded multiple layers (material properties: $E_i, h_i$ ) with halfspace equivalent thickness $h_{II}$ on substructure

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$M_{\text{long I}} = M_{\text{long II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.28})$$

$$M_{\text{lat I}} = M_{\text{lat II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.29})$$

$$M_{\text{long } 2} = M_{\text{long II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.30})$$

$$M_{\text{lat } 2} = M_{\text{lat II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.31})$$

$$\sigma_{\text{long } 1} = 6 \times \frac{M_{\text{long } 1}}{h_1^2}, \sigma_{\text{long } 2} = 6 \times \frac{M_{\text{long } 2}}{h_2^2} [\text{N} / \text{mm}^2] \quad (\text{B.32})$$

$$\sigma_{\text{lat } 1} = 6 \times \frac{M_{\text{lat } 1}}{h_1^2}, \sigma_{\text{lat } 2} = 6 \times \frac{M_{\text{lat } 2}}{h_2^2} [\text{N} / \text{mm}^2] \quad (\text{B.33})$$

where

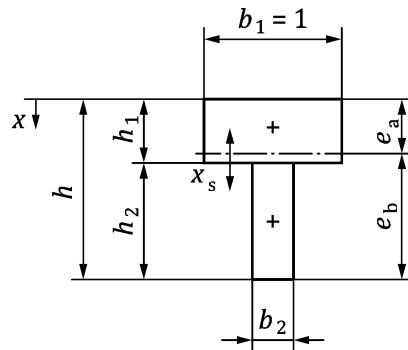
$M_{\text{long II}}$  and  $M_{\text{lat II}}$  are the bending moments [Nmm] calculated using the Westergaard approach;

$h_1$  and  $h_2$  are the thicknesses [mm] of layers 1 and 2;

$E_1$  and  $E_2$  are the Young's moduli [N/mm<sup>2</sup>] of layers 1 and 2.

**System variant III: fully bonded multiple layers (material properties:  $E_i$ ,  $h_i$ ) with half-space equivalent thickness  $h_{\text{III}}$  on substructure**

Model: T-beam (with equivalent stiffness  $E=E_1$ )



#### Key

- $h$  thickness of both bonded layers
- $h_1$  thickness of first layer
- $h_2$  thickness of second layer
- $b_1$  reference width of first layer
- $b_2$  equivalent width of second layer
- $e_a$  distance between surface of first layer and neutral axis of the model T-beam
- $e_b$  distance between neutral axis of the model T-beam and the bottom of the second layer
- $x_s$  distance between any point of interest and the neutral axis of the model T-beam

**Figure B.11 — T-beam model for equivalent layer stiffness**

NOTE The formulae are applicable in case the neutral axis of the composite section is inside the top layer.

$$b_2 = \frac{E_2}{E_1} \quad (B.34)$$

$$e_a = \frac{\sum (A_i \times x_i)}{\sum A_i} [\text{mm}], \text{ where: } A_1 = 1 \times h_1 [\text{mm}^2] \text{ and } A_2 = b_2 \times h_2 [\text{mm}^2] \quad (B.35)$$

$$e_b = h - e_a [\text{mm}], \text{ where } h = h_1 + h_2 [\text{mm}] \quad (B.36)$$

The moment of inertia of a T-beam [mm<sup>4</sup> per mm]:

$$I = \sum \left( I_i + A_i \times x_{s,i}^2 \right) [\text{mm}^4] \text{ where } I_i = \frac{b_i \times h_i^3}{12} [\text{mm}^4] \quad (B.37)$$

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$\sigma_{\text{long1,top}} = \frac{M_{\text{long III}}}{I} \times e_a [\text{N} / \text{mm}^2] \quad (B.38)$$

$$\sigma_{\text{long1,bottom}} = \frac{M_{\text{long III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.39)$$

$$\sigma_{\text{long2,top}} = b_2 \times \frac{M_{\text{long III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.40)$$

$$\sigma_{\text{long2,bottom}} = b_2 \times \frac{M_{\text{long III}}}{I} \times e_b [\text{N} / \text{mm}^2] \quad (B.41)$$

$$\sigma_{\text{lat1,top}} = \frac{M_{\text{lat III}}}{I} \times e_a [\text{N} / \text{mm}^2] \quad (B.42)$$

$$\sigma_{\text{lat1,bottom}} = \frac{M_{\text{lat III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.43)$$

$$\sigma_{\text{lat2,top}} = b_2 \times \frac{M_{\text{lat III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.44)$$

$$\sigma_{\text{lat2,bottom}} = b_2 \times \frac{M_{\text{lat III}}}{I} \times e_b [\text{N} / \text{mm}^2] \quad (B.45)$$

where

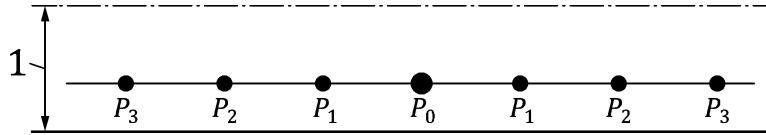
$M_{\text{long III}}$  and  $M_{\text{lat III}}$  are the bending moments [Nmm] calculated using the Westergaard approach;

$h_1$  and  $h_2$  are the thicknesses [mm] of layers 1 and 2;

$E_1$  and  $E_2$  are the Young's moduli [N/mm<sup>2</sup>] of layers 1 and 2.

### B.1.6 Beam on Winkler foundation (Zimmermann): Longitudinal bending moment and tensile stress due to rail seat loads

#### B.1.6.1 General



Key  
1 1/2 width

Figure B.12 — Beam model, rail seat configuration

#### B.1.6.2 Longitudinal bending moment $M_{\text{long I,II,III}}$ [Nmm] due to rail seat loads

##### B.1.6.2.1 General

Longitudinal bending moment  $M_{\text{long I,II,III}}$  [Nmm] activated by the traffic load at rail seat location 0 may be computed from the bending moment  $M_0$  I,II,III [Nmm], due to rail seat load  $P_0$  [N], and the longitudinal bending moment  $M_{\text{long neigh}}$  [Nmm], due to neighbouring rail seat load  $P_j$  [N].

$$M_{\text{long I,II,III}} = M_0 \text{ I,II,III} + M_{\text{long neigh}}$$

##### B.1.6.2.2 Bending moment $M_0$ I,II,III [Nmm] due to rail seat load $P_0$ [N]

Using Zimmermann calculation, the bending moment in the infinitely long concrete beam due to load  $P_0$  is given below. Since the system is symmetrical in transverse direction, any half-side of the slab may be used.

$$M_{0 \text{ I,II,III}} = \frac{P_0 \times L_{\text{el}}}{4} \quad (\text{B.46})$$

The maximum rail seat load  $P_0$  [N] is calculated according to Annex A.

The elastic length  $L_{\text{el}}$  [mm] of the beam is given below:

$$L_{\text{el}} = \sqrt[4]{\frac{4 \times E_1 \times I_B}{b_B \times k}} \quad (\text{B.47})$$

where

$E_1$  is the Young's modulus [N/mm<sup>2</sup>] of concrete beam [N/mm<sup>2</sup>];

$k$  is the bedding modulus [N/mm<sup>3</sup>].

The moment of inertia of the beam  $I_B$  [mm<sup>4</sup>] is given below:

$$I_B = \frac{b_B \times h^3}{12} \quad (\text{B.48})$$

where

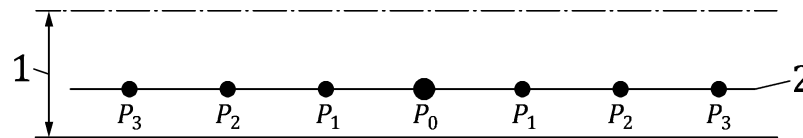
$b_B$  is half of slab width [mm];

$h$  is thickness [mm] of system  $h_I$  or  $h_{II}$  or  $h_{III}$ .

#### B.1.6.2.3 Longitudinal bending moment $M_{\text{long neigh}}$ [Nmm] due to neighbouring rail seat load $P_j$ [N]

Check on additional longitudinal bending moments in the concrete beam shall be made using the Zimmermann approach. Depending on the elastic length  $L_{el}$ , neighbouring rail seat loads increase the bending moment at rail seat location 0.

Using the approach, a beam of width equal to half of the slab width (any half, left or right, may be used) and thickness  $h_1$  (as earlier defined) is assumed to be loaded vertically by the rail seat loads  $P_j$ , see Figure B.13.



Key

- 1 1/2 width
- 2 left/right rail

Figure B.13 — Rail seat configuration

Bending moment at 0 due to neighbouring loads  $P_j$  at distances  $x_j$  from the reference load  $P_0$ :

$$M_{\text{long neigh}} = \frac{L_{el}}{4} \times \sum (P_j \times \mu_j) [\text{Nmm}] \quad (\text{B.49})$$

$$\mu_j = \frac{-\sin \xi_j + \cos \xi_j}{e^{\xi_j}} \quad (\text{B.50})$$

$$\text{with : } \xi_j = x_j / L_{el} \quad (\text{B.51})$$

where

$L_{el}$  is the elastic length [mm] of the beam system;

$x_j$  is the distance [mm] between the rail seat 0 and the rail seat  $j$ ;

$P_j$  are the rail seat loads [N] calculated according to Annex A.

Only positive  $\mu_j$  shall be taken into account.

#### B.1.6.3 Longitudinal bending tensile stress $\sigma_{\text{long}}$ [N/mm<sup>2</sup>] due to traffic load

**System variant I: single layer (thickness  $h_I$ ) on substructure**

Longitudinal bending tensile stress at bottom of slab:

$$\sigma_{\text{long I}} = \frac{6 \times M_{\text{long I}}}{\frac{B}{2} \times h_1^2} \quad (\text{B.52})$$

where

$B$  is the width [mm] of the slab of the single layer.

**System variant II: unbonded multiple layers (material properties:  $E_i$ ,  $h_i$ ) with halfspace equivalent thickness  $h_{\text{II}}$  on substructure**

Longitudinal bending tensile stresses in layers 1 and 2:

$$M_{\text{long I}} = M_{\text{long II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.53})$$

$$M_{\text{long 2}} = M_{\text{long II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.54})$$

$$\sigma_{\text{long I}} = 6 \times \frac{M_{\text{long I}}}{\frac{B_1}{2} \times h_1^2}, \sigma_{\text{long 2}} = 6 \times \frac{M_{\text{long 2}}}{\frac{B_2}{2} \times h_2^2} [\text{N/mm}^2] \quad (\text{B.55})$$

where

$M_{\text{long II}}$  is the bending moment [Nmm] calculated using the Zimmermann approach;

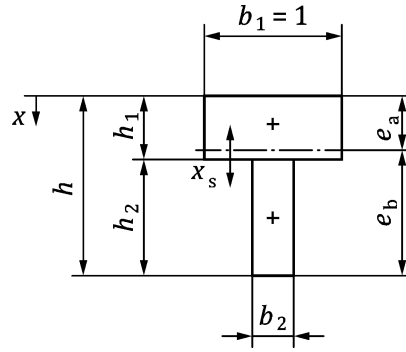
$h_1$  and  $h_2$  are the thicknesses [mm] of layers 1 and 2;

$E_1$  and  $E_2$  are the Young's moduli [N/mm<sup>2</sup>] of layers 1 and 2;

$B_1$  and  $B_2$  are the width [mm] of the slabs of layers 1 and 2.

**System variant III: fully bonded multiple layers (material properties:  $E_i$ ,  $h_i$ ) with half-space equivalent thickness  $h_{\text{III}}$  on substructure**

Longitudinal and lateral bending tensile stresses in layers 1 and 2.



**Key**

- $h$  thickness of both bonded layers
- $h_1$  thickness of first layer
- $h_2$  thickness of second layer
- $b_1$  reference width of first layer
- $b_2$  equivalent width of second layer
- $e_a$  distance between surface of first layer and neutral axis of the model T-beam
- $e_b$  neutral axis of the model T-beam and the bottom of the second layer
- $x_s$  distance between any point of interest and the neutral axis of the model T-beam

**Figure B.14 — Model T-beam (with equivalent stiffness  $E = E_1$ )**

$$b_2 = \frac{E_2}{E_1} \quad (B.56)$$

$$e_a = \frac{\sum (A_i \times x_i)}{\sum A_i} \text{ mm, where: } A_1 = b_1 \times h_1 \text{ [mm}^2\text{] and } A_2 = b_2 \times h_2 \text{ [mm}^2\text{]} \quad (B.57)$$

$$e_b = h - e_a \text{ [mm], where } h = h_1 + h_2 \text{ [mm]} \quad (B.58)$$

The moment of inertia of a T-beam [mm<sup>4</sup> per mm]:

$$I = \sum \left( I_i + A_i \times x_{s,i}^2 \right) \text{ [mm}^4\text{]}, \text{ where } I_i = \frac{b_i \times h_i^3}{12} \text{ [mm}^4\text{]} \quad (B.59)$$

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$\sigma_{\text{long 1,top}} = \frac{M_{\text{long III}}}{\frac{B_1}{2} \times I} \times (e_a) \text{ [N / mm}^2\text{]}, \sigma_{\text{long 1,bottom}} = \frac{M_{\text{long III}}}{\frac{B_1}{2} \times I} \times (h_1 - e_a) \text{ [N / mm}^2\text{]} \quad (B.60)$$

$$\sigma_{\text{long 2,top}} = b_2 \times \frac{M_{\text{long III}}}{\frac{B_2}{2} \times I} \times (h_1 - e_a) \text{ [N / mm}^2\text{]}; \sigma_{\text{long 2,bottom}} = b_2 \times \frac{M_{\text{long III}}}{\frac{B_2}{2} \times I} \times e_b \text{ [N / mm}^2\text{]} \quad (B.61)$$



where

- $M_{\text{long III}}$  are the bending moment [Nmm] calculated by the Zimmermann approach;
- $h_1$  and  $h_2$  are the thicknesses [mm] of layers 1 and 2;
- $E_1$  and  $E_2$  are the Young's moduli [N/mm<sup>2</sup>] of layers 1 and 2;
- $B_1$  and  $B_2$  are the width [mm] of the slabs of layers 1 and 2.

### B.1.7 Critical longitudinal bending tensile stress

The critical longitudinal bending tensile stress activated at the position of rail seat load  $P_0$  shall be determined. The critical longitudinal bending tensile stress will be the greatest of:

The longitudinal bending tensile stress  $\sigma_{\text{long}}$  [N/mm<sup>2</sup>] calculated using the longitudinal bending moment  $M_{\text{long I}}$  or  $M_{\text{long II}}$  or  $M_{\text{long III}}$  at slab interior (Westergaard approach), and

the longitudinal bending tensile stress  $\sigma_{\text{long}}$  [N/mm<sup>2</sup>] calculated using the longitudinal bending moment  $M_{\text{long I}}$  or  $M_{\text{long II}}$  or  $M_{\text{long III}}$  in half width of the pavement (Zimmermann approach).

### B.1.8 Critical lateral bending tensile stress

The critical lateral bending tensile stress activated at the position of rail seat load  $P_0$  is the lateral bending tensile stress  $\sigma_{\text{lat}}$  [N/mm<sup>2</sup>] calculated using the bending moment  $M_{\text{lat I}}$  or  $M_{\text{lat II}}$  or  $M_{\text{lat III}}$  at slab interior by Westergaard approach.

## B.2 Stresses in concrete slab/pavement due to thermal impact

### B.2.1 General

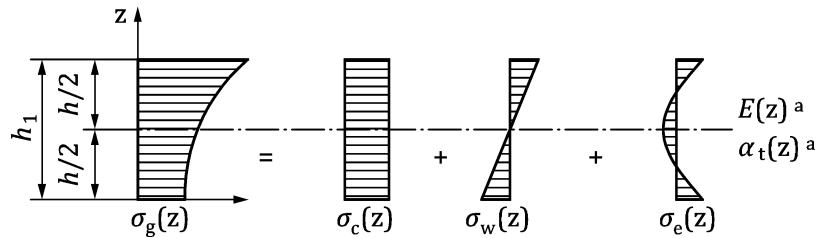
Temperature changes in the subsystem concrete slab/pavement cause deformation and stresses. With respect to potential deformations, the stresses (tensile stresses are decisive) acting along the thickness  $h$  should be separated into constant  $\sigma_c$ , linear  $\sigma_w$  and residual  $\sigma_e$  stresses. More specifically:

Constant stresses  $\sigma_c$ : Cooling ( $\Delta T$ ) causes changes of slab-length and slab-width (the longest one is decisive) and/or constant stresses  $\sigma_c$ , due to friction at the bottom interface, and/or longitudinal restraint by longitudinal reinforcement (CRCP). In this case, relevant for the design are the characteristic tensile stresses activated by cooling, during winter time, see B.2.2.

Linear stresses  $\sigma_w$ : Heating causes bending of the slab and remaining warping stresses  $\sigma_w$ , due to linear temperature gradient  $\Delta t$  dependent on the slab dimensions and the support conditions. In this case, relevant for the design are the characteristic bending tensile stresses activated by heating, during summer time, see B.2.3.

Residual stresses  $\sigma_e$ : Residual stresses are not converted into deformations. This stress partition at bottom of the slab is typically in compression and not relevant for design.

The highest stress level calculated by B.2.2 and B.2.3 shall be used as constant stress level to determine the stress limit for traffic load.



NOTE  $a = \text{constant}$

**Figure B.15 — Stresses in concrete slab/pavement due to thermal impact**

## **B.2.2 Constant stresses $\sigma_c$ due to temperature changes $\Delta T$ acting in concrete slabs or pavements**

### **B.2.2.1 Jointed Plain Concrete Pavements (JPCP)**

Constant stresses  $\sigma_c$  due to temperature changes  $\Delta T$  are not relevant in case slab length / joint spacing is not exceeding 5 m.

### **B.2.2.2 Continuously Reinforced Concrete Pavements (CRCP) or Jointed Reinforced Concrete Pavements (JRCP)**

For reducing the crack width in the concrete slab/pavement continuously reinforced concrete pavements with controlled (JRCP) or random cracking (CRCP) shall be used. A percentage of longitudinal reinforcement equivalent to 0,8 % to 0,9 % and diameter  $\varnothing$  18 mm to 20 mm is recommended for CRCP in order to achieve fine cracks (crack width < 0,5 mm and crack spacing < 5 m) and to avoid merging cracks. JRCP is typically using smaller amount of reinforcement e.g. 0,4 % to 0,5 %.