A.2 Rail break gap for track on a bridge, with conventional fastenings

For conventional CWR on a bridge, the calculation shall be undertaken considering the interaction effects between the bridge and the rail. A computer simulation based on a model according to EN 1991-2:2003, 6.5.4.4 may be used. However, if the track configuration is of a design used elsewhere in the railway network, it is unlikely that unacceptable rail break gaps will open up on bridges except in two specific cases:

— sliding fastenings are used to reduce rail stresses due to track-bridge interaction effects;

— the normal rail stress limits have been exceeded and the track engineer and certification authority have given a derogation permitting those limits to be increased on the bridge.

In either of these cases a special calculation is required to ensure that permitted rail break gaps are not exceeded.

If the rail break occurs in CWR on a bridge, the total width of the rail break gap is the result of the gap given by the thermal expansion of the restrained rail and of the displacements and expansion of the bridge. Since the system is nonlinear, the effects have to be calculated and superposed nonlinearly.

In principle rail breaks are detected quickly and the effect of frequently applied braking forces does not need to be take into account. Since rail breaks are more likely to occur in winter under high tension stresses, the calculation should be done for the following load cases:

- maximum negative bridge temperature variation ΔT_B (minimum bridge temperature) generally $\Delta T_B = -30$ K

- maximum negative rail temperature variation ΔT_R (minimum rail temperature) generally $\Delta T_R = -50$ K

For the calculation it can be assumed, that the characteristics of the unloaded track are considered and that both rails of a track break together (conservative assumption). On bridges with more than one track, the break only has to be considered in one track.

Breaks are most likely to occur at the position where the theoretical maximum tension stresses occur (generally over bridge joint).

The nonlinear characteristics of the longitudinal resistance of the track have to be considered.

The following steps have to be undertaken for the decisive point of the rail on the bridge:

1) Load case ΔT_B and ΔT_R

↓ nonlinear calculation

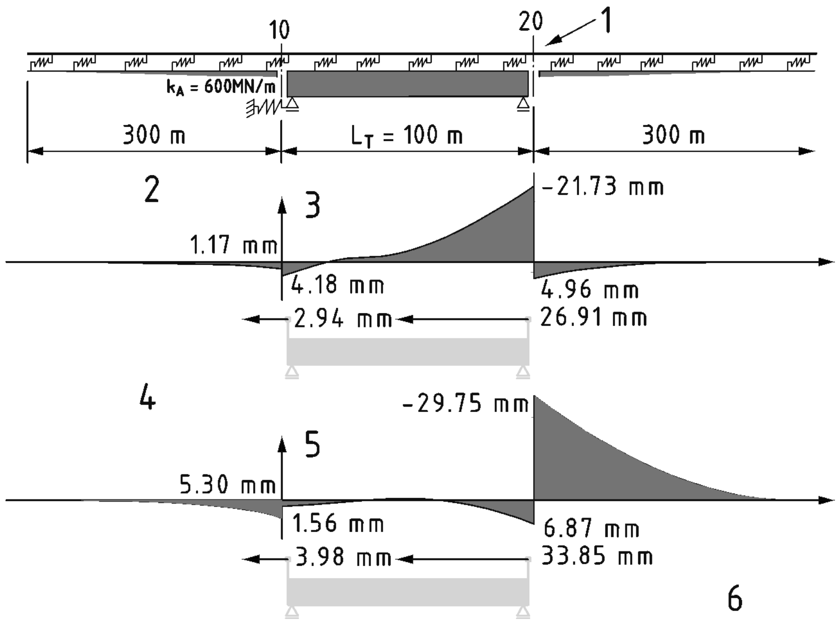
2) Modelling of the rail break

↓ nonlinear calculation

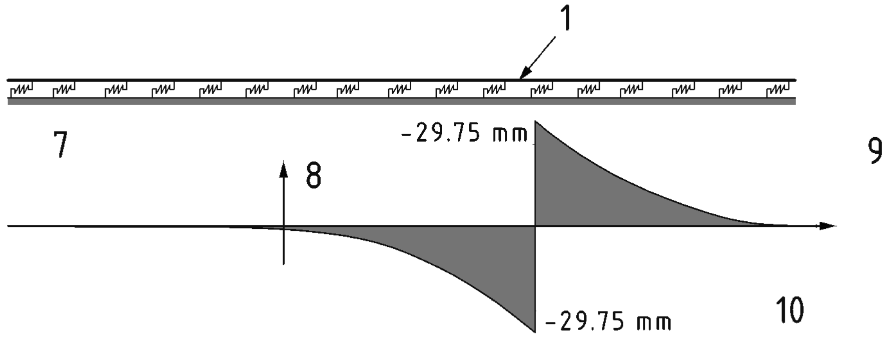
3) Evaluation of the gap width

An example of the calculation procedure is presented in the following (for the selected static system see also the case studies in Annex C):

a) rail break on bridge



b) Rail break on embankment (comparison)



Key

1 rail break
2 state after $\Delta T_{\text{Bridge}} = -30\,\text{K}$ and $\Delta T_{\text{Rail}} = -50\,\text{K}$
3 $U_{\text{Rail/substructure}}$
4 state after rail break
5 $U_{\text{Rail/substructure}}$
6 rail break gap 70,48 mm
7 state after $\Delta T_{\text{Rail}} = -50\,\text{K}$ and rail break
8 $U_{\text{Rail/substructure}}$
9 rail break gap 59,51 mm
10 Remark: free thermal expansion of the bridge:

$L_T = 100\,\text{m}$

$\alpha^{\text{th}} = 1{,}2 \times 10^{-5}\ 1/\text{K}$

$\Delta T_{\text{Bridge}} = -30\,\text{K}$

$\Delta L_T = 36\,\text{mm}$

Figure A.2 — Rail break on a single span bridge

As can be seen in Figure A.2, the rail break gap occurring at axis 20 of the bridge (70,48 mm) is significantly dependent on the interaction effects between the rail and bridge. A nonlinear calculation should be performed in order to obtain realistic values for the rail break gap.

A.3 Rail break gap for track with sliding (ZLR) fastenings

If there is a rail break within a length of track, *LS*, for which *LR* is zero (i.e. a length of track with "mobile rail" or ZLR fastenings) the restrained rail at either side of that length will pull back by the amount calculated in A.1 above, but there will be an additional gap width, $w_{\text{gap,ZLR}}$, given by the thermal expansion properties of the unrestrained length which is given by

$$w_{gap} = w_{gap,CWR} + w_{gap,ZLR}$$

A.4 Limiting values of rail break gap

The maximum permitted value of w_{gap} may be specified by the Infrastructure Manager or the individual project specification. In the absence of any other information, for conventional railways it is recommended that a value of w_{gap} = 90 mm should be used. This value is based on the German specification DB Ril 820.2040, 2(4).

Similarly, for light rail systems it is recommended that a value of w_{gap} = 75 mm should be used. This value is based on the value given in a number of individual project specifications issued by US based engineering consultants in recent years.

Annex B
(informative)

Algebraic studies of longitudinal track characteristics

B.1 Algebraic representations of behaviour

B.1.1 Sliding action

Fundamental to the longitudinal interaction between the track and the bridge is the sliding action that can occur between the rail and the structure. Elastic-plastic sliding mechanisms are not often met within structural mechanics, so the procedures for evaluating their effects are unfamiliar. The resulting structural behaviour is also unfamiliar. A section on this topic is included here (B.1.1 to B.1.7) because there are no text books or published papers that set out the basic behaviour that needs to be understood when designing a bridge to support a railway. Fryba addresses the topic at some length [Fryba, L. Dynamics of Railway Bridges. Thomas Telford Ltd., London 1996] but he only addresses the case of an elastic k-function. Models based on values of the track resistance to sliding k within the elastic range do not reflect what happens in practice, where relatively small deflections are needed to initiate the onset of plastic behaviour.

In this section a number of concepts are identified, and given names and which are shown in italics, such as *k*-function. Different calculation methods are discussed. The analytical studies undertaken result in algebraic expressions for the parameters which are needed to apply this standard. These expressions give insights into the interaction behaviour, and it is shown how some of them are used in two spreadsheets (see B.2) which derive all the parameters needed for the checks.

The behaviour and analysis methods described relate to the case with continuous rails, with no rail joints. Bridges may have one or two spans between abutments, or they may be very long viaducts which are broken down into a succession of decks, separated by structure joints. Each deck may be subdivided into two, three or four spans, but its structure is continuous over the full length of the deck. It is always assumed that there is a structure joint between an abutment and a deck. Even if the deck is fixed to the abutment, the abutment has some flexibility, and this flexibility has a significant effect on the behaviour. It is assumed that the top of the embankments which lead up to the abutments at both ends are fixed longitudinally. In reality there will be some flexibility within the body of the embankments, but it has little effect on the parameters being calculated.

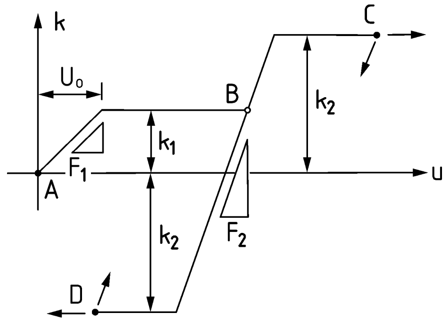
It is longitudinal actions that are discussed and analysed. This results in values for the Additional Stress, and for the longitudinal force onto the substructure. These are the parameters which need to be checked. There are four actions which contribute to these parameters: change in temperature of the deck, end-rotations of the decks due to Temperature Difference, end-rotations due to vertical train loads, and longitudinal sway due to braking and traction forces. The stresses and movements depend on the sequence in which the loads are applied and move. Current practice uses the LM71 load model (see 6.3.2 of EN 1991-2:2003) to model the effect of the train, with the load positioned on the bridge where it gives the most severe effects (see 6.8.1 in EN 1991-2:2003). As explained in 10.2.5, LM71 cannot be used to model the moving train using step-by-step behaviour adjusting the position of LM71, so simplifying assumptions are made. In these studies it is assumed that a seasonal temperature change occurs first, followed by Temperature Difference, and then the train load and the braking and traction effects occur simultaneously. It is shown that the Temperature Difference effect can be combined with the effect of the seasonal temperature change, so there are two loading stages, and two analysis stages that relate to them. There is also a combined analysis stage, in which all the effects are combined.

B.1.2 The *k*-function

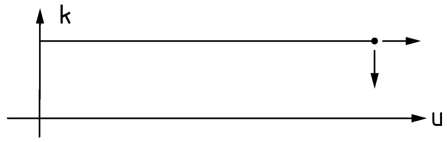
In UIC774-3R and EN 1991-2:2003 an elastic-plastic model is used to describe the *k*-function. This has been maintained in the recommended amendments to EN 1991-2 in Annex E of this report. The parameters k and u_0 are used to define this simplified function, as outlined in 5.2.2 above. In order to have a better insight into track-bridge interaction behaviour it is worth considering models in which the structure may be represented in more detail but in which the *k*-function is simplified even further. In this section k is used as a generic term for the force transfer between track and deck or subgrade. k_T is defined as the plastic value of k for the temperature case. A derived parameter F_1 is used for the elastic flexibility of the system, where $F_1 = u_0/k_T$. In the case of ballasted track some of the flexibility is in the ballast and some in the fastening. F, the generic symbol for either F_1 or F_2, can be expressed as $F_B + F_F$, where F_B and F_F are the flexibility of the ballast and the fastening.

The fully-plastic model of the *k*-function is a simplification which assumes that $u > u_0$ at all locations on the bridge. This is a good approximation for long multi-span bridges with low values of u_0 e.g. long bridges with ballastless track with quite stiff fastening systems. Use of this further simplification enables algebraic expressions to be derived. The expressions give insights into the behaviour but it is not intended that they should be used except for approximate evaluations of parameters. The gap model is also used because it can simplify calculations with little loss of accuracy. All three functions are shown in Figure B.1.

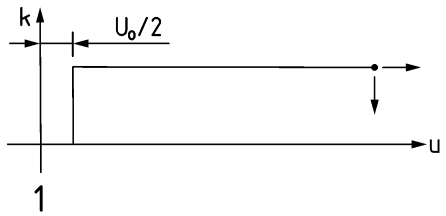
When the track over the joint is loaded, or the ballast is frozen, the load - displacement parameters of the track change. Figure B.1 shows the elastic-plastic model before and after a change at Point B. Temperature change takes it from A to B. k_T and F_T then change to k_E and F_E. In this analysis, u_0 is only used to describe the case under the first stage of loading (temperature). $u_{0,E}$ the value of u_0 to be used in the analysis for the second stage of loading (braking and traction and vertical train loading), is sometimes specified but it is not used directly. After k changes value, or a change in the direction of movement, the change in u which initiates slip is described as $F_2 \times k'$, where k', the effective k, is the change in the plateau value of k. If the next movement is in the same direction as the temperature change, then $k' = k_E - k_T$ and the *k*-function progresses from B to C. If it is the opposite direction $k' = k_E + k_T$ and it progresses from B to D. After C or D it will follow one of the arrows depending on whether the next movement is in the same direction as the previous movement, or the opposite direction. The first stage of loading may cease before u reaches $u_{0,T}$. In this case the point B lies on the elastic part of the diagram. If the second movement is in the same direction as the first, $k' = k_E - k_1$, where k_1 is the value of k after the first stage. If it is in the opposite direction $k' = -k_E - k_1$. In both cases $u_{0,E} = k' \times F_E$.



a) Elastic plastic model



b) Fully plastic model



c) Gap model

Figure B.1 — *k*-functions

In the case of the loaded track it is not clear how the enhanced *k* value is sustained under a moving train. A fastening can carry a large shear force when an axle is above it, but when the axle moves away its shear capacity reverts to the shear capacity of the unloaded track. In the case of a loaded track two calculations are recommended, firstly using k_T and secondly using k_E. It is likely that k_E governs peak rail stresses and k_T governs the sway and anchor force.

The elastic-plastic behaviour of the *k*-function is inherently nonlinear, so the effects of different actions cannot be added directly without, possibly, introducing significant conservatism in the resulting designs. The spreadsheets use iterative methods which allow the effects to be combined correctly.

B.1.3 Temperature change

B.1.3.1 General

The Additional Stress is the additional axial stress in the rail in the vicinity of a bridge compared with the coexisting axial stress in the rail away from the bridge. If there is no joint in the rail, the temperature stresses due to changes in the temperature of the rail are the same near the bridge and away from the bridge. It follows that changes in the temperature of the rail do not contribute to the Additional Stress. However changes in the temperature of the bridge cause movements in the structure joints between decks which, in turn, mobilize *k*-forces in the track and generate Additional Stress in the rail.

B.1.3.2 Algebraic studies under changing seasonal temperatures

Algebraic expressions are derived below for several parameters. Each time an expression is derived it is evaluated using the following typical values:

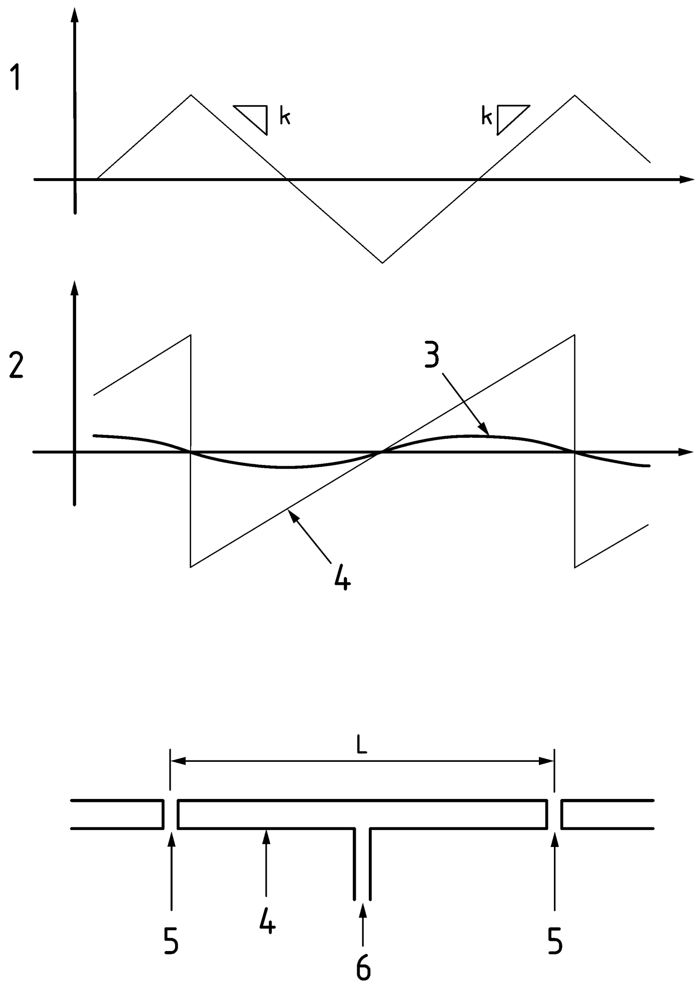
E		= 205 000 MPa
A	= 2 × 60/7 850	= 0,015 3 m^2
k_T		= 0,020 MN/m
k_E		= 0,060 MN/m
u_0		= 0,002 5 m
L_J		= 90 m
α		= 12e-6
T		= 35K
K		= 300 MN/m

Study 1. A repeating deck within a long viaduct.

Using the fully-plastic model.

A typical, repeating deck lies within a long length of identical decks, all anchored longitudinally at their midpoint. This case can be used to explore the minimum costs needed for a long, repetitive viaduct. The fully-plastic k-function is used to derive simple expressions. Figure B.2 shows plots of the longitudinal track forces and the longitudinal displacements of both the track and the deck under an annual temperature change. Note that the force diagram necessarily subtends equal areas above and below the axis because there can be no net expansion or contraction of the rail. The direction of slip changes at every deck joint, and every midpoint. Where the direction changes the plot of the rail displacement changes from convex to concave.

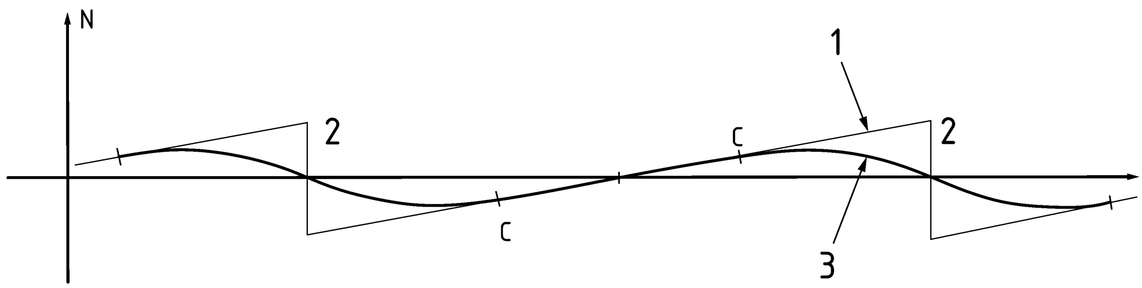
In a region of slip (i.e. where $u > u_0$) $dN/dx = k_T$. Longitudinal strain $dv/dx = N/EA$, so $N = EA\, dv/dx$, $k_T = EA\, d^2v/dx^2$, $d^2v/dx^2 = k''$, where $k'' = k_T/EA$. If $dv/dx = 0$ at $x = 0$, then $v = k''x^2/2$. This shows that each length of the rail displacement diagram is a parabola with a constant curvature of k'', so the radius of curvature $R = 1/k''$. The gradient of the diagram is the axial strain in the rail. The absolute maximum temperature stress, which occurs both at the deck joints and at the mid-point, M, of the deck = $k_T L/4A$ = 29,4 MPa. There appears to be an immediate paradox. The stress due to the temperature change is predicted to be independent of the magnitude of the temperature change. This can be explained. Under small temperature changes slip is limited to regions close to the deck joints. The displacement plot for this case is shown in Figure B.3. Over the middle length, CC, of the deck the rail displacement diagram is captured by the deck displacement diagram. As the temperature change increases the curved sections of the displacement plot extend until they meet, when slip is fully developed – i.e $u > u_0$ at every rail fastening position - and the temperature change $T_T = k_T L/4EA\alpha$ = 12,0 °C. This is called the Tangent temperature change because the deck displacement plot is tangential to the rail displacement plot at the midpoint. As T increases further the gradient of the deck plot increases but the rail plot and rail stress is unchanged.



Key

1 track force
2 displacement
3 rail
4 deck
5 deck joint
6 anchor pier

Figure B.2 — Plots of track force and displacement for a repeating deck



Key

1 deck

2 joint

3 rail

Figure B.3 — Plot of track displacement with partial slip

Using the gap model.

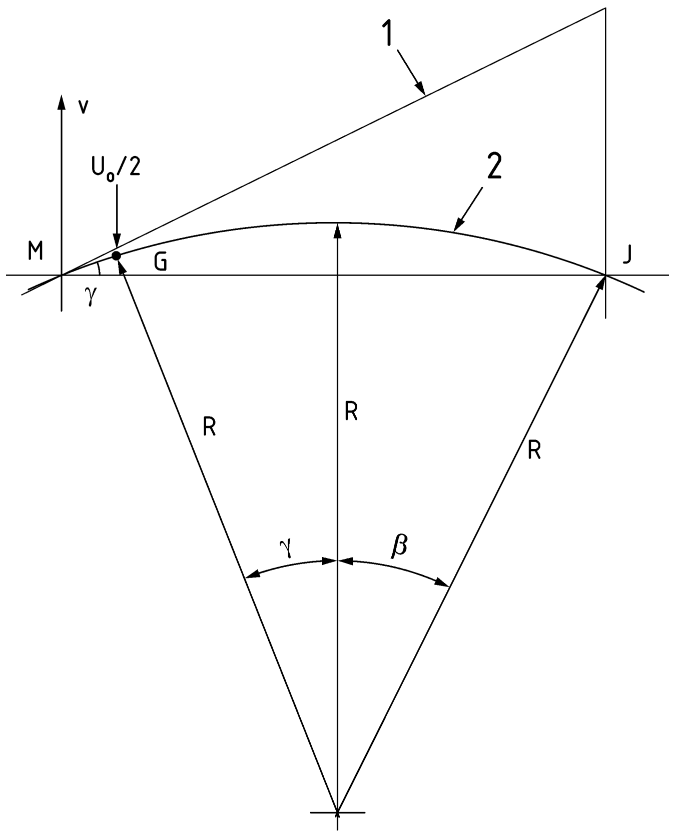
The gap model gives a good approximation of the elastic-plastic behaviour. It requires the solution of a cubic equation, which is solved with a formula. It gives accurate results over a wide range of parameters. Figure B.4 shows the displacement plots over half the length of the deck. γ is the gradient of the rail plot within the gap. β is the negative gradient of the rail plot at the end of the deck. The difference between the two plots is $u_0/2$ at $(u_0/2)/(\alpha T— \gamma)$ from M. Two formulae, (B.1) and (B.2), can be derived from Figure B.4.

$$\frac{\frac{u_0}{2}}{\alpha T - \gamma} + R \sin \gamma + R \sin \beta = L/2 \tag{B.1}$$

$$\gamma \frac{L}{2} - \frac{k"\left(\frac{L}{2} - \frac{\frac{u_0}{2}}{\alpha T - \gamma}\right)^2}{2} = 0 \tag{B.2}$$

If $p = \alpha T - \gamma$, so $\gamma = \alpha T - p$

$$\alpha TL - pL - k"\left(\frac{L}{2} - \frac{u_0}{2p}\right)^2 = 0$$



Key

1 deck

2 rail

Figure B.4 — A half deck with plots of *N* and *v* using the gap model

$$\alpha TL - pL - k''\left(\frac{L^2}{4} - L\frac{u_0}{2p} + \frac{u_0^2}{4p^2}\right) = 0$$

$$-Lp^3 + \left(\alpha TL - \frac{k'' L^2}{4}\right)p^2 + k'' L u_0 \frac{p}{2} - \frac{k'' u_0^2}{4} = 0$$

$$p^3 + \left(-\alpha T + \frac{k'' L}{4}\right)p^2 + \left(-k''\frac{u_0}{2}\right)p + \left(\frac{k'' u_0^2}{4L}\right) = 0$$

The cubic may be solved using the method set out in Louis Pipes, Applied Mathematics for Engineers and Scientists, McGraw-Hill Kogakusha, 1946. International Student Edition, page 118.

$p^3 + A_2p^2 + A_1p + A_0 = 0$

$q = + A_2^2/3 - A_1$

$r = - A_0 + A_1A_2/3 - 2/27A_2^3$