ANNEX A (normative)

Membrane theory stresses in shells

A.1 General

A.1.1 Action effects and resistances

The action effects or resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the action effect or resistance when characteristic values of the actions, geometric parameters and material properties are adopted.

A.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

F_x	axial load applied to the cylinder
F_z	axial load applied to a cone
M	global bending moment applied to the complete cylinder (not to be confused with the moment per unit width in the shell wall m)
M_t	global torque applied to the complete cylinder
V	global transverse shear applied to the complete cylinder

Roman lower case letters

g	unit weight of the material of the shell
p_n	distributed normal pressure
p_x	distributed axial traction on cylinder wall

Greek lower case letters

ϕ	meridional slope angle
σ_x	axial or meridional membrane stress ($= n_x/t$)
σ_θ	circumferential membrane stress ($= n_\theta/t$)
τ	membrane shear stress ($= n_{x\theta}/t$)

A.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 2.3 and 5.2.2.

(2) For these expressions to be strictly valid, the boundary conditions for cylinders should be taken as radially free at both ends, axially supported at one end, and rotationally free at both ends.

(3) For these expressions to be strictly valid for cones, the applied loading should match a membrane stress state in the shell and the boundary conditions should be taken as free to displace normal to the shell at both ends and meridionally supported at one end.

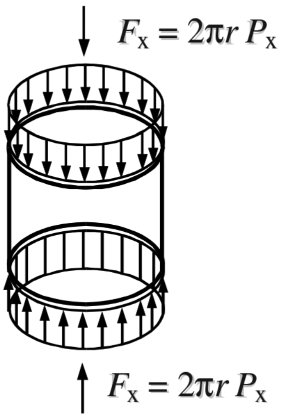
(4) For truncated cones, the boundary conditions should be understood to include components of loading transverse to the shell wall, so that the combined stress resultant introduced into the shell is solely in the direction of the shell meridian.

A.1.4 Sign convention

(1) The sign convention for stresses σ should be taken everywhere as tension positive, though some of the figures illustrate cases in which the external load is applied in the opposite sense.

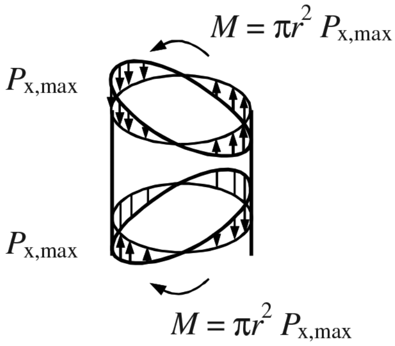
A.2 Unstiffened cylindrical shells

A.2.1 Uniform axial load



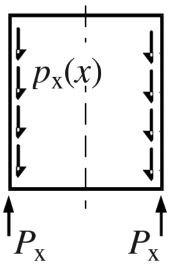
$$\sigma_x = -\frac{F_x}{2\pi r t}$$

A.2.2 Axial load from global bending



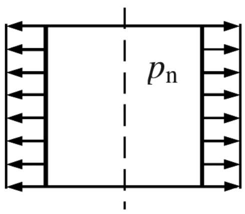
$$\sigma_x = \pm\frac{M}{\pi r^2 t}$$

A.2.3 Friction load



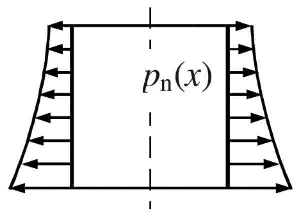
$$\sigma_x = -\frac{1}{t}\int_0^l p_x \cdot dx$$

A.2.4 Uniform internal pressure



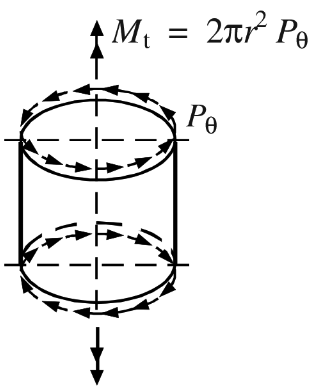
$$\sigma_\theta = p_n \cdot \frac{r}{t}$$

A.2.5 Variable internal pressure



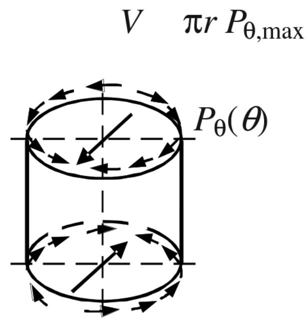
$$\sigma_\theta(x) = p_n(x) \cdot \frac{r}{t}$$

A.2.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

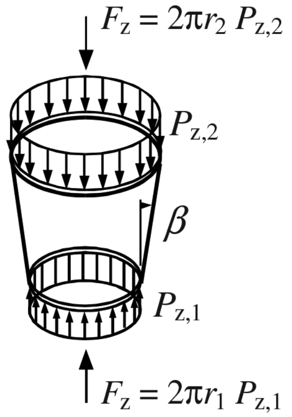
A.2.7 Sinusoidal shear from transverse force



$$\tau_{max} = \pm\frac{V}{\pi r t}$$

A.3 Unstiffened conical shells

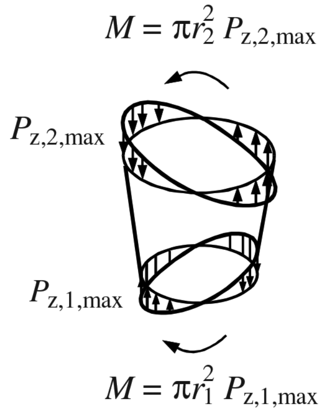
A.3.1 Uniform axial load



$$\sigma_x = -\frac{F_z}{2\pi r t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

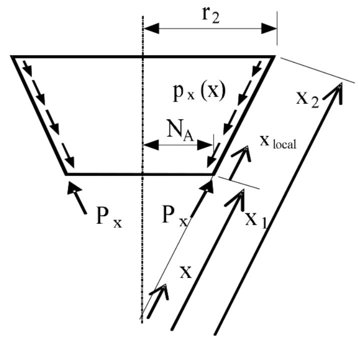
A.3.2 Axial load from global bending



$$\sigma_{x,\max} = \pm \frac{M}{\pi r^2 t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

A.3.3 Friction load

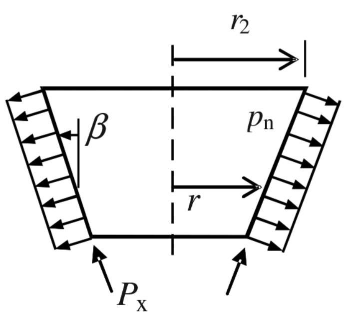


$$x_1 = \frac{r_1}{\sin\beta} \qquad x_2 = \frac{r_2}{\sin\beta}$$

$$\sigma_{x1} = -\frac{1}{x_1 t} \int_{x_1}^{x_2} p_x x \cdot dx$$

$$\sigma_\theta = 0$$

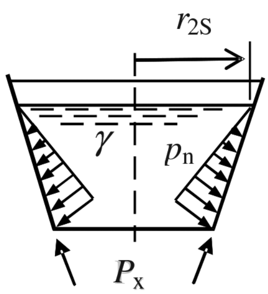
A.3.4 Uniform internal Pressure



$$\sigma_x = -p_n \frac{r}{2t \cdot \cos\beta} \left[\left(\frac{r_2}{r} \right)^2 \right]$$

$$\sigma_\theta = p_n \frac{r}{t \cdot \cos\beta}$$

A.3.5 Linearly varying internal pressure

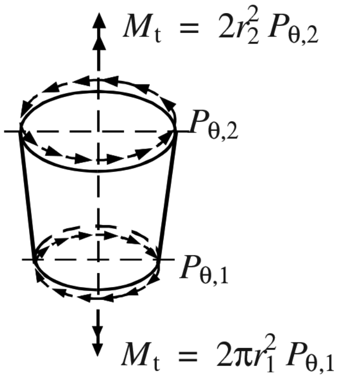


r_{2S} is the radius at the fluid surface

$$\sigma_x = -\frac{\gamma r}{t \cdot \sin\beta} \left\{ \frac{r_{2s}}{6} \left[\left(\frac{r_{2s}}{r} \right)^2 - 3 \right] + \frac{r}{3} \right\}$$

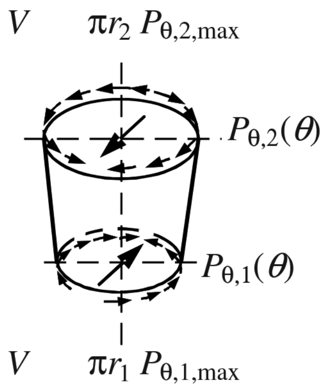
$$\sigma_\theta = +\frac{\gamma r}{t \cdot \sin\beta} (r_{2s} - r)$$

A.3.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

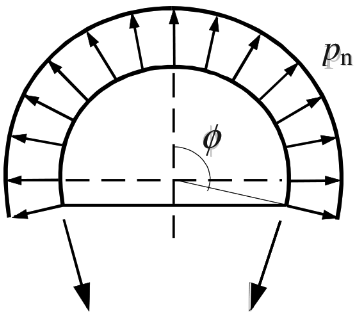
A.3.7 Sinusoidal shear from transverse force



$$\tau_{max} = \pm\frac{V}{\pi r t}$$

A.4 Unstiffened spherical shells

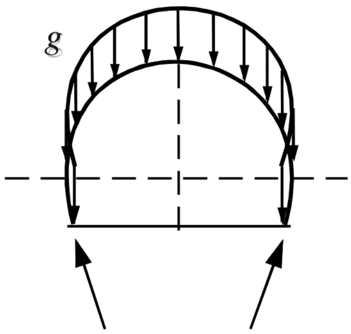
A.4.1 Uniform internal pressure



$$\sigma_x = \frac{p_n r}{2t}$$

$$\sigma_\theta = \frac{p_n r}{2t}$$

A.4.2 Uniform self-weight load



$$\sigma_x = -\frac{gr}{t}\left(\frac{1}{1+\cos\phi}\right)$$

$$\sigma_\theta = -\frac{gr}{t}\left(\cos\phi - \frac{1}{1+\cos\phi}\right)$$

ANNEX B (normative)

A1⟩ Additional expressions for plastic reference resistances ⟨A1

B.1 General

B.1.1 Resistances

The resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the resistance when characteristic values of the geometric parameters and material properties are adopted.

B.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

A_r	cross-sectional area of a ring
P_R	characteristic value of small deflection theory plastic mechanism resistance

Roman lower case letters

b	thickness of a ring
ℓ	effective length of shell which acts with a ring
r	radius of the cylinder
s_e	dimensionless von Mises equivalent stress parameter
s_m	dimensionless combined stress parameter
s_x	dimensionless axial stress parameter
s_θ	dimensionless circumferential stress parameter

Subscripts

r	relating to a ring
R	resistance

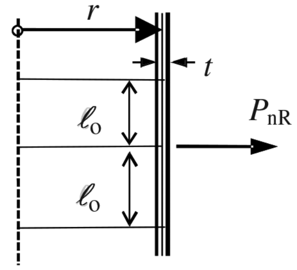
B.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 5.2.2.

(2) The term “clamped” should be taken to refer to BC1r and the term “pinned” to refer to BC2f.

B.2 Unstiffened cylindrical shells

B.2.1 Cylinder: Radial line load



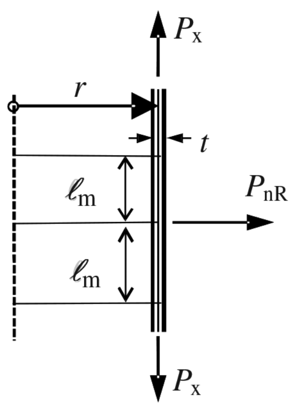
Reference quantities:

$$\ell_o = 0{,}975\sqrt{rt}$$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_0} = f_y \frac{t}{r}$$

B.2.2 Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad \ell_o = 0{,}975\sqrt{rt}$$

Range of applicability:

$$-1 \le s_x \le +1$$

Dependent parameters:

If $P_n > 0$ (outward) then: $A = + s_x - 1{,}50$

If $Pn < 0$ (inward) then: $A = - s_x - 1{,}50$

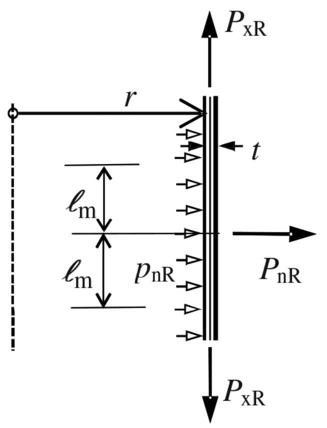
$$s_m = A + \sqrt{A^2 + 4(1 - s_x^2}$$

If $s_x \neq 0$ then: $\ell_m = s_m \ell_0$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_m} = f_y \frac{t}{r}$$

B.2.3 Cylinder: Radial line load, constant internal pressure and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad s_\theta = \frac{P_n}{f_y} \cdot \frac{r}{t}$$

$$\ell_o = 0{,}975\sqrt{rt} \qquad s_e = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$$

Range of applicability:

$$-1 \leq s_x \leq +1 \qquad -1 \leq s_\theta \leq +1$$

Dependent parameters:

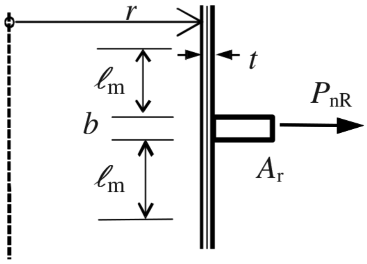
Outward directed ring load $P_n > 0$		Inward directed ring load $P_n < 0$	
Condition	Expressions	Condition	Expressions
$s_e < 1{,}00$ and $s_\theta \leq 0{,}975$	$A = +s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1\text{-}s_\theta} \right)$	$s_e < 1{,}00$ and $s_\theta \geq -0{,}975$	$A = -s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1\text{+}s_\theta} \right)$
$s_e = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0.0$	$s_e = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0.0$

The plastic resistance is given by (P_n and p_n always positive outwards):

$$\frac{P_{nR}}{2\ell_m} + p_n = f_y \frac{t}{r}$$

B.3 Ring stiffened cylindrical shells

B.3.1 Ring stiffened Cylinder: Radial line load

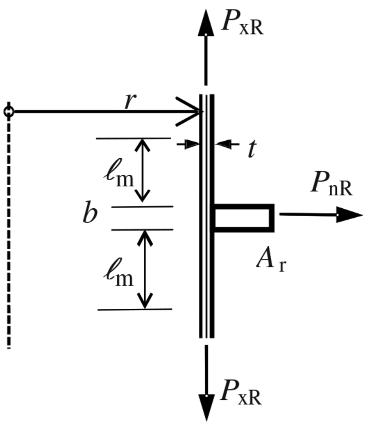


The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)^t}{r} \right)$$

$$\ell_m = \ell_0 = 0,975\sqrt{rt}$$

B.3.2 Ring stiffened Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{p_x}{f_y t} \qquad \ell_0 = 0,975\sqrt{rt}$$

Range of applicability:

$$-1 \leq s_x \leq +1$$

Dependent parameters:

If $P_n > 0$ then: $A = + s_x - 1,50$

If $P_{\mathrm{n}} < 0$ then: $A = -s_{\mathrm{x}} - 1{,}50$

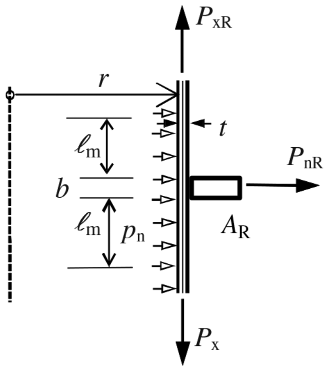
$$s_{\mathrm{m}} = A + \sqrt{A^2 + 4(1 - s_{\mathrm{x}}^2}$$

If $s_{\mathrm{x}} \neq 0$ then: $\ell_{\mathrm{m}} = s_{\mathrm{m}} \ell_0$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$p_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)^t}{r} \right)$$

B.3.3 Ring stiffened cylinder: Radial line load, constant internal pressure and axial load



Reference quantities:

$$s_x = \frac{p_x}{f_y t} s_x = \frac{p_x}{f_y t} \qquad s_\theta = \frac{p_n}{f_y} \cdot \frac{r}{t}$$

$$\ell_0 = 0.975\sqrt{rt} \qquad s^{\mathrm{e}} \sqrt{s_\theta^2 + s_{\mathrm{x}}^2 - s_x s_\theta}$$

Range of applicability:

$$-1 \leq s_{\mathrm{x}} \leq +1 \qquad -1 \leq s_\theta \leq +1$$

Dependent parameters:

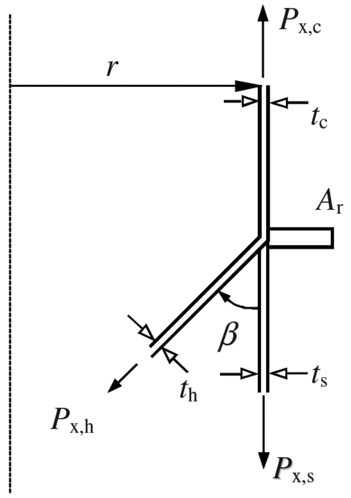
Outward directed ring load $P_{\mathrm{n}} > 0$		Inward directed ring load $P_{\mathrm{n}} < 0$	
Condition	Expressions	Condition	Expressions
$s_{\mathrm{e}} < 1{,}00$ and $s_\theta \leq 0{,}975$	$A = +s_{\mathrm{x}} - 2s_\theta - 1{,}50$ $s_{\mathrm{m}} = A + \sqrt{A^2 + 4(1 - s_{\mathrm{e}}^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 - s_\theta} \right)$	$s_{\mathrm{e}} < 1{,}00$ and $s_\theta \geq -0{,}975$	$A = -s_{\mathrm{x}} - 2s_\theta - 1{,}50$ $s_{\mathrm{m}} = A + \sqrt{A^2 + 4(1 - s_{\mathrm{e}}^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 + s_\theta} \right)$
$s_{\mathrm{e}} = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_{\mathrm{m}} = 0{,}0$	$s_{\mathrm{e}} = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_{\mathrm{m}} = 0{,}0$

The plastic resistance is given by (P_n and p_n always positive outwards):

$$p_{nR} + p_n(b + 2\ell_m) = f_y\left(\frac{A_r + (b + 2\ell_m)t}{r}\right)$$

B.4 Junctions between shells

B.4.1 Junction under meridional loading only (simplified)



Range of applicability:

$$t_c^2 \le t_s^2 + t_h^2 \qquad |P_{xs}| << t_s f_y, |P_{xh}| << t_h f_y, \text{ and } |P_{xc}| << t_c fy,$$

Dependent parameters:

$$\eta = \sqrt{\frac{t_c^2}{t_s^2 + t_h^2}} \qquad \psi_s = \psi_h = 0,7 + 0,6\eta^2 - 0,3\eta^3$$

For the cylinder $\quad \ell_{oc} = 0,975\sqrt{rt_c}$

For the skirt $\quad \ell_{os} = 0,975\ \psi_s\sqrt{rt_s}$

For the conical segment $\quad \ell_{oh} = 0,975\ \psi_h\sqrt{\frac{rt_h}{\cos\beta}}$

The plastic resistance is given by:

$$p_{xhR}\, r\sin\beta = f_y(A_r + \ell_{oc}t_c + \ell_{os}t_s + \ell_{oh}t_h)$$