## Annex A

(normative)

## Design of components by calculation

## A.1 Introduction

The design of components by calculation is based on the concept of EN 1992-1-1 as far as possible. Some modifications have been made in order to take into account the specific material properties/behaviour of LAC.

Design values to be used can be determined according to the provisions of this annex, using the relevant partial safety factors.

Values of the properties can be determined on the basis of calculation and can be given as declared values in CEmarking, according to the relevant provisions in Annex ZA. The declared values are based on one of the three methods given in 3.3.3.2 (a).of Guidance Paper L.

## A.2 General

(1)P Depending on the character of the individual clauses, distinction is made in this annex between Principles and Application Rules.

(2)P The Principles comprise

- general statements and definitions for which there is no alternative and
- requirements and analytical models for which no alternative is permitted unless specifically stated.
- (3)P In this annex the Principles are marked by a number in brackets followed by the letter P.

(4)P Application Rules are generally recognized rules which follow the Principles and satisfy their requirements.

(5)P In this annex Application Rules are marked by a number in brackets not followed by the letter P.

(6)P Design by calculation shall be based upon documented material parameters, using analytical methods that adequately describe the structural behaviour of the components.

(7)P Nominal dimensions, including those tolerances, which lead to the most unfavourable result, shall be used in the calculations.

(8) A.4 to A.7 describe general calculation procedures that may be used for components subjected to bending, shear, or axial compression. A.8 gives specific rules applicable to the different types of components.

(9)P More detailed analyses may be carried out on the basis of the design assumptions given in A.4.1.

NOTE The design of the components requires also a thorough consideration of the overall stability and stiffness of the entire building. If the stiffness and the stability of the structure are not obvious, analytical verification of the stability of both horizontal and vertical stiffening members is necessary. In the case of high deformability of the stiffening members allowance should be made for the influence of their deformations on the effects of actions (second order effects).

## A.3 Partial safety factors

Partial safety factors are determined according to national application documents.

NOTE An example is given in informative Annex C.

# A.4 Ultimate limit state – Design for bending and combined bending and axial compression

### A.4.1 Design assumptions

- (1)P The design assumptions in a) to j) given below shall be used.
- a) Plane sections remain plane.
- b) The strain in the reinforcement is the same as that in the surrounding LAC.
- c) The flexural strength of LAC is generally ignored. It may be taken into account in wall components for bending as a result of wind actions in combination with vertical actions.
- d) The stresses in LAC in compression are derived from the design stress-strain diagram given in Figure A.1.
- e) The stresses in the reinforcement are derived from the design stress-strain diagram given in Figure A.2.
- f) Deformations and second order effects are calculated using the mean values of the material properties (such as  $E_{cm}$  according to 4.2.6). For cross-sectional design, the design values of the material properties are used.
- g) For cross-sections subject to centrally loaded longitudinal compression, the compressive strain of LAC is limited to 0,002.
- h) For cross-sections not fully in compression, the limiting compressive strain of LAC is given by Equation (A.1). In intermediate situations, the strain distribution is defined by assuming that the compressive strain is 0,002 at a level ( $\varepsilon_{cu}$  0,002)/ $\varepsilon_{cu}$  of the height of the section from the most compressed face.
- i) The adoption of the above assumptions leads to the range of potential strain diagrams shown in Figure A.3.
- j) The effect of longitudinal reinforcement present in the compression zone is not taken into account when calculating the axial loadbearing capacity, unless the reinforcing bars are sufficiently restrained to the principal reinforcement in the tension zone, e.g. by stirrups.
- NOTE The use of the flexural strength in bending is described in the national application document as indicated in A.6.3.3.3.

## A.4.2 Stress-strain diagram for LAC

(1)P The stress-strain diagram for LAC for cross-sectional design is shown in Figure A.1, where the ultimate compressive strain  $\varepsilon_{cu}$  is given by the equation

$$\varepsilon_{cu} = 0,003 \ 5 \ \eta_1 \ge 0,002$$
 (A.1)

where

 $\eta_1 = 0.40 + 0.60 \rho / 2 200$  (see Equation (2a));

ho is the dry density of LAC, in kilograms per cubic metre.

The design value of the compressive strength of LAC is defined by

$$f_{\rm cd} = f_{\rm ck} / \gamma_{\rm C}$$

(A.2)

#### where

- $f_{cd}$  is the design value of the compressive strength of LAC, in megapascals;
- $f_{ck}$  is the characteristic compressive strength of LAC, in megapascals;
- $_{\mathcal{R}}$  is the partial safety factor for concrete (also for LAC) according to A.3.

NOTE 1 The value of  $\chi_c$  for use in a country can be found in its national application document. The recommended values for use are given in Table C.1.

(2)P Giving allowance for long term effects on the compressive strength of LAC the design strength is multiplied by the coefficient  $\alpha$ .

NOTE 2 The value of  $\alpha$  for use in a country can be found in the national application document. The recommended value of  $\alpha$  is 0,85. In cases where the compression zone decreases in width in the direction of the extreme compression fibre  $\alpha$  is recommended to be reduced to 0,80.



Key

1 idealized diagram

2 design diagram

#### Figure A.1 — Bi-linear stress-strain diagram for LAC in compression for cross-sectional design

#### A.4.3 Stress-strain diagram for reinforcement steel

(1)P The design value  $f_{yd}$  of the yield strength of reinforcement steel is defined by:

$$f_{\rm yd} = f_{\rm yk}/\gamma_{\rm S}$$

where

 $f_{yk}$  is the characteristic yield strength of reinforcement steel;

 $\gamma_{S}$  is the partial safety factor for reinforcement steel.

NOTE The value of  $\gamma_{s}$  for use in a country can be found in its national application document. Recommended values are given in Table C.1.

(2) The design stress-strain diagram for reinforcement steel is given in Figure A.2. Other established stressstrain diagrams may be used.  $E_s$  is the modulus of elasticity of reinforcement steel (e.g.  $2 \times 10^5$  MPa).

(A.3)



#### Key

- 1 idealized diagram
- 2 design diagram





#### Key

1 neutral axis

The value of  $\beta$  is equal to  $(\varepsilon_{cu} - 0,002)/\varepsilon_{cu}$ , where  $\varepsilon_{cu}$  is calculated from Equation (A.1).

#### Figure A.3 — Possible strain diagrams in the ultimate limit state

### A.5 Ultimate limit state induced by shear

# A.5.1 Shear design for components predominantly under transverse load not requiring shear reinforcement

(1)P The following conditions shall be met:

$V_{Ed} < V_{Rd1}$	(A.4a)
$V_{\rm Ed}$ < $V_{\rm Rd2}$	(A.4b)

where

56

 $V_{Ed}$  is the design shear force in the section;

 $V_{Rd1}$  is the design shear resistance of a component without shear reinforcement;

*V*<sub>Rd2</sub> is the design shear crushing resistance (maximum shear force that can be carried without crushing of the notional concrete compressive struts), which is given by Equation (A.14).

The design shear force does not need to be checked at a distance of less than d/2 from the support. A reduction of concentrated loads near the support is not allowed.

NOTE 1 The value of  $V_{Rd1}$  can be estimated by Equation (A.5) or Equation (A.10) as specified in the national application document.

(2) The design shear resistance  $V_{Rd1}$  may be estimated as:

$$V_{\rm Rd1} = C_{\rm Rd} k \cdot \eta_1 (100 \ \rho_1 f_{\rm ck})^{1/3} b_{\rm w} d \tag{A.5a}$$

with a minimum of

$$V_{\text{Rd1}} = v_{\text{mind}} b_{\text{w}} d$$
 (A.5b)

#### where

 $C_{Rd}$  is a parameter for shear capacity;

- k is a factor for the shear strength, see Equation (A.8);
- $\eta_1$  is a coefficient according to Equation (A.9);
- $\rho_1$  is the reinforcement ratio  $\rho_1 = A_{s1}/(b_w d) \leq 0.02$ ;
- $f_{ck}$  is the characteristic compressive strength of LAC, in megapascals;
- A<sub>s1</sub> is the cross-sectional area of the longitudinal reinforcement, anchored as described in A.9, in square millimetres;
- $b_{\rm w}$  is the minimum width of the cross-section over the effective depth, in millimetres;
- *d* is the effective depth of the cross-section, in millimetres;

 $v_{mind}$  is the minimum shear strength of LAC, in megapascals.

NOTE 2 The values of  $C_{rd}$  and  $v_{mind}$  can be found in the national application document. Recommended values are:

$C_{\rm Rd} = 0,145 / \gamma_{\rm C}$	(A.6)
$v_{\rm mind} = 0.03 \cdot k^{3/2} \cdot f_{\rm ck}^{1/2}$	(A.7)
$k = 1 + (200/d)^{\frac{1}{2}} \le 2,0$	(A.8)
$\eta_1 = 0.40 + 0.60 \rho/2 \ 200$	(A.9)

#### where

- $\gamma_{C}$  is the partial safety factor for LAC;
- $f_{ck}$  is the characteristic compressive strength of LAC, in megapascals;
- $\rho$  is the dry density of LAC, in kilograms per cubic metre.
- (3) The design shear resistance  $V_{Rd1}$  may be estimated as:

 $V_{\rm Rd1} = \tau_{\rm Rd} k (1, 2 + 40 \rho_1) b_{\rm w} d$ 

(A.10)

57

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$\tau_{\rm Rd} = \tau_{\rm Rk} / \gamma_{\rm C}$	(A.11)
$\tau_{\rm Rk} = 0,125 f_{\rm t,fik}$	(A.12)
$k = 1,6 - d/1000 \ge 1$	(A.13)

#### where

 $\tau_{Rk}$  is the characteristic shear strength of LAC, in megapascals;

k is a factor for the shear strength, see Equation (A.13);

 $\rho_1$  is the reinforcement ratio  $\rho_1 = A_{s1}/(b_w d) \leq 0.02;$ 

 $\tau_{Rd}$  is the design shear strength of LAC, in megapascals;

- $\chi_{C}$  is the partial safety factor for LAC;
- A<sub>s1</sub> is the cross-sectional area of the longitudinal reinforcement, anchored as described in A.9, in square millimetres;
- $b_{\rm w}$  is the minimum width of the cross-section over the effective depth, in millimetres;
- *d* is the effective depth of the cross-section, in millimetres;
- $f_{t,flk}$  is the characteristic flexural strength of LAC (see 4.2.4), in megapascals.

(4) The design shear crushing resistance  $V_{Rd2}$  is given by:

$$V_{\rm Rd2} = 0.5 \ \eta_1 b_{\rm w} z \ v f_{\rm ck} / \gamma_{\rm C}$$

where

- $\eta_1$  is a coefficient according to Equation (A.9);
- $b_{\rm w}$  is the minimum width of the cross-section over the effective depth, in millimetres;
- *z* is the height of the shear zone, which is taken as the distance between the centres of the compression zone and the longitudinal reinforcement (lever arm of internal forces), in metres. This may be set to 0.9d in normal designs, but not more than the height ( $h_w$ ) of the shear reinforcement;

(A.14)

- *d* is the effective depth of the cross-section, in metres;
- $h_{\rm w}$  is the height of the shear reinforcement, in metres;
- v is the efficiency factor, which shall be taken as 0,6;
- $f_{ck}$  is the characteristic compressive strength of LAC, in megapascals;
- $\gamma_{\rm C}$  is the partial safety factor for LAC.

## A.5.2 Shear design for components predominantly under transverse load requiring shear reinforcement

(1)P The following conditions shall be met:

$V_{\rm Ed} < V_{\rm Rd3}$	(A.15a)
$V_{\rm Ed}$ < $V_{\rm Rd2}$	(A.15b)

where

58

 $V_{Ed}$  is the design shear force in the section;

*V*<sub>Rd2</sub> is the design shear crushing resistance (maximum shear force that can be carried without crushing of the notional concrete compressive struts), which is given by Equation (A.14);

 $V_{\rm Rd3}$  is the design shear resistance of the component with shear reinforcement.

The design shear force does not need to be checked at a distance of less than d/2 from the support. A reduction of concentrated loads near the support is not allowed.

NOTE 1 The value of  $V_{Rd3}$  can be estimated by Equation (A.16) or Equation (A.17) as specified in the national application document.

(2)P For beams with vertical shear reinforcement (ladder-type), the shear resistance  $V_{Rd3}$  is

$$V_{\text{Rd3}} = (A_{\text{sw}}/s) \cdot z f_{\text{ywd}} \cdot \cot\theta$$

where

 $A_{sw}$  is the cross-sectional area of shear reinforcement, in square millimetres;

- *s* is the spacing of the shear reinforcement bars, in metres;
- *z* is the height of the shear zone, which is taken as the distance between the centres of the compression zone and the longitudinal reinforcement (lever arm of internal forces), in metres. This may be set to 0.9d in normal designs, but not more than  $h_w$ ;
- $h_{\rm w}$  is the height of the shear reinforcement, in metres;
- $f_{ywd}$  is the design yield strength of the shear reinforcement ( $f_{ywd} = f_{ywk}/\gamma_s$ ), in megapascals;
- $f_{\text{vwk}}$  is the characteristic value of the yield strength of the shear reinforcement, in megapascals;
- $\gamma_{\rm S}$  is the partial safety factor for reinforcement steel;
- $\theta$  is the angle of the compression struts.

The shear reinforcement shall fulfil the minimum requirements in (4).

The angle  $\theta$  of the compression struts shall be limited.

NOTE 2 The limiting values of  $\cot\theta$  can be found in the national application document. The recommended limits are  $1 \le \cot\theta \le 2,5$ .

(3)P For beams with vertical shear reinforcement (ladder-type), the shear resistance  $V_{Rd3}$  is

$$V_{\text{Rd3}} = V_{\text{Rd1}} + V_{\text{wd}}$$

where  $V_{Rd1}$  is the design shear resistance of a component without shear reinforcement given by Equation (A.10) and  $V_{wd}$  is estimated by Equation (A.18) as

$$V_{wd} = 0.8 (A_{sw} / s) z f_{ywd}$$

where

 $A_{sw}$  is the area of shear reinforcement, in square millimetres;

- *s* is the spacing of the shear reinforcement bars, in metres;
- *z* is the height of the shear zone, which is taken as the distance between the centres of the compression zone and the longitudinal reinforcement (lever arm of internal forces), in metres. This may be set to 0.9d in normal designs, but not more than  $h_{w}$ .

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(A.16)

(A.17)

(A.18)

- *d* is the effective depth of the cross-section, in metres;
- $h_{\rm w}$  is the height of the shear reinforcement, in metres;
- $f_{ywd}$  is the design yield strength of the shear reinforcement ( $f_{ywd} = f_{ywk}/\gamma_s$ ), in megapascals;
- $f_{ywk}$  is the characteristic value of the yield strength of the shear reinforcement, in megapascals;
- $\gamma_{\rm S}$  is the partial safety factor for reinforcement steel.

The shear reinforcement shall fulfil the minimum requirements in (4).

(4) The minimum shear reinforcement ratio  $\rho_w$  for beams shall be at least

$$\rho_{\rm w,min} = 0.08 f_{\rm ck}^{\frac{1}{2}} / f_{\rm yk}$$
 (A.19)

#### where

$$\rho_{\rm w} = A_{\rm sw} / \left( s \cdot b_{\rm w} \right) \tag{A.20}$$

#### and where

- $f_{ck}$  is the characteristic compressive strength of LAC, in megapascals;
- $f_{yk}$  is the characteristic yield strength of the reinforcement steel, in megapascals;
- $A_{sw}$  is the cross-sectional area of shear reinforcement within length s, in square millimetres;
- *s* is the spacing of the shear reinforcement measured along the longitudinal axis of the member, in millimetres;
- $b_{\rm w}$  is the width of the web of the beam, in millimetres.

Using reinforcement steel with yield strength  $f_{yk}$  = 500 MPa will lead to the following minimum percentages of shear reinforcement shown in Table A.1:

Table A.1 — Minimum percentages of	shear reinforcement $\rho_{w,min}$	for reinforcement steel w	vith f <sub>vk</sub> = 500 MPa
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Characteristic compressive strength of LAC	how,min
∫ <sub>ck</sub> MPa	%
2	0,023
4	0,032
6	0,039
8	0,045
10	0,051
12	0,055
15	0,062
20	0,072
25	0,080

The shear reinforcement shall be vertical and welded to the longitudinal reinforcement. Welding and anchorage shall be verified according to A.9.

(5) The value of the characteristic value of the yield strength of the shear reinforcement  $f_{ywk}$  should be limited.

NOTE 3 The limiting value of  $f_{ywk}$  can be found in the national application document. The recommended value is  $f_{ywk} \leq$  400 MPa.

## A.5.3 Shear design for components under predominantly longitudinal compression forces, e.g. walls and piers

(1)P Only the portions of the cross-section which remain without tensile stresses under the relevant load combination in the ultimate limit state are considered to be capable of resisting concrete stresses due to shear.

(2)P The design shear resistance is:

$$V_{\text{Rd4}} = \tau_{\text{Rd}} b_{\text{w}} x / 1,5$$

$$\tau_{\text{Rd}} = 0,125 f_{\text{t,fik}} / \gamma_{\text{C}}$$
(A.21)
(A.22)

where

 $\tau_{Rd}$  is the basic shear strength of LAC, in megapascals;

- $b_{\rm w}$  ~ is the minimum width of the section in the compression zone, in millimetres;
- x is the neutral axis depth  $x \le h$ ; calculated using first order theory, in millimetres;
- $f_{t,flk}$  is the characteristic flexural strength of LAC (see 4.3.4), in megapascals;
- *h* is the overall depth of the cross-section, in millimetres;
- $\gamma_{\rm C}$  is the partial safety factor for LAC.

In cross-sections where the zone of decompression in the ultimate limit state extends further than to the centre of the cross-section it shall be verified that:

$$N_{\rm Sd} / V_{\rm Sd} \ge 2,0$$

where

- $N_{Sd}$  is the design axial compression force in the section;
- $V_{Sd}$  is the design shear force in the section.

### A.6 Ultimate limit state induced by structural deformation (buckling)

#### A.6.1 General

(1)P When determining the loadbearing capacity of slender LAC components subjected to compression or combined bending and compression, account shall be taken of the effects of structural deformation and eccentricities occurring perpendicular to the plane of the components and their influence on the buckling of the components.

(2) Two methods are given in A.6.2 and A.6.3. They are both suitable for designing vertical loadbearing LAC components which can be classified as slender isolated columns or walls and are mainly loaded by a central or eccentric longitudinal load and possibly also by a transverse load (e.g. horizontal wind load, earth pressure).

(3)P The slenderness ratio S of the components shall not exceed the values indicated in Figure A.4,

where

- *S* is the slenderness ratio;  $S = l_0/i_w \le 121$ ;
- *l*<sub>0</sub> is the buckling length of the component, in metres. The buckling length (effective height with respect to buckling) will depend on the support conditions, as shown in Table A.2;

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(A.23)

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- $i_w$  is the radius of inertia in the direction of the weak axis ( $i_w = 0,289h$  for rectangular solid cross-sections), in metres;
- *h* is the overall thickness of the component, in metres;
- ho is the mean value of the dry density of the LAC, in kilograms per cubic metre.



#### Key

- *S* slenderness ratio
- ρ mean dry density of LAC

#### Figure A.4 — Limits of slenderness ratio S of loadbearing walls and piers

NOTE The use of the design methods according to the different methods of this subclause (A.6.2, A.6.3 or A.6.4) in a country is specified in the national application document. The national application document can limit the use of these components for different structural applications such that e.g. different minimum thicknesses are required or different maximum slenderness limits are specified for certain structural applications.

#### A.6.2 Method based on the Euler formula

(1)P The resisting design axial compression force  $N_{Rd}$  shall be determined as the loadbearing capacity of that part of the cross-section, which can be regarded as centrally loaded, i.e.:

$$N_{\rm Rd} = k_{\rm s} \cdot \alpha f_{\rm cd} A_{\rm c} \tag{A.24}$$

where

- $k_{\rm s}$  is the column factor according to Equation (A.25);
- $\alpha$  is the long term factor according to A.4.2;
- $f_{cd}$  is the design value of the compressive strength of LAC;
- $A_{c}$  is the area of the compression zone of the cross-section;