

$$L_{el} = \sqrt[4]{\frac{E_1 \times h^3}{12 \times (1 - \mu^2) \times k}} \quad (B.15)$$

where

- E_1 is the Young's modulus [N/mm²] of slab/pavement – 1st layer;
- h is the thickness of the slab/pavement [mm] or h_I or h_{II} or h_{III} ;
- μ is the Poisson's ratio of the slab/pavement material;
- k is the bedding modulus [N/mm³].

Longitudinal and lateral bending moments (M_{long} and M_{lat}) activated by adjacent rail seat loads P_j at distance x_j from reference load P_0 may be computed from the radial and tangential moments $M_{r,t}$. Radial and tangential moments activated by neighbouring rail seat loads are given by:

$$M_{j-r,t} = \lambda_{r,t} \times P_j \quad (B.16)$$

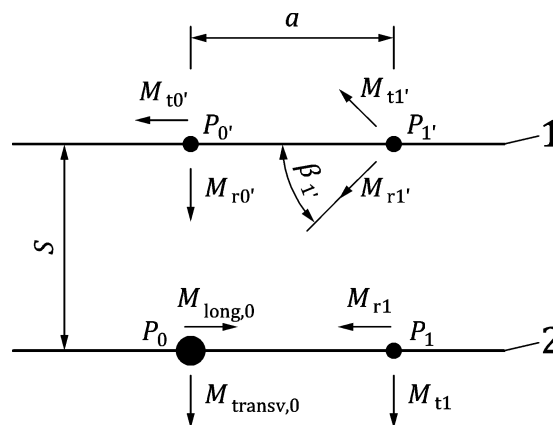
The rail seat loads P_j [N] are calculated according to Annex A and to compute $\lambda_{r,t}$ Formulae (B.17) and (B.18) shall be used:

$$\lambda_{r,j} = 0,160 - 0,284 \times \left(\frac{x_j}{L_{el}}\right) + 0,157 \times \left(\frac{x_j}{L_{el}}\right)^2 - 0,036 \times \left(\frac{x_j}{L_{el}}\right)^3 + 0,003 \times \left(\frac{x_j}{L_{el}}\right)^4 \quad (B.17)$$

$$\lambda_{t,j} = 0,2499 - 0,3511 \times \left(\frac{x_j}{L_{el}}\right) + 0,2034 \times \left(\frac{x_j}{L_{el}}\right)^2 - 0,0536 \times \left(\frac{x_j}{L_{el}}\right)^3 + 0,0052 \times \left(\frac{x_j}{L_{el}}\right)^4 \quad (B.18)$$

As an example, the above computation for the Figure B.9 of the influence of neighboring loads P_1 , P_1' , and P_0' on P_0 , are shown in Figure B.10 and Formulae (B.17) and (B.18).

For each rail seat location of interest, the radial and tangential bending moments (M_r and M_t) shall be computed.



Key

- 1 left rail
- 2 right rail

Figure B.10 — Example of configuration of radial and tangential bending moments

Right rail:

$$P1: M_{\text{long},1} = \lambda_{r,1} \times P_1 \quad (\text{B.19})$$

$$M_{\text{lat},1} = \lambda_{t,1} \times P_1 \quad (\text{B.20})$$

Left rail:

$$P0: M_{\text{long},0'} = \lambda_{r,0'} \times P_{0'} \quad (\text{B.21})$$

$$M_{\text{lat},0'} = \lambda_{r,0'} \times P_{0'} \quad (\text{B.22})$$

$$P1: M_{\text{long},1'} = \lambda_{r,1'} \times P_{1'} + 0,5 \times (\lambda_{t,1'} \times P_{1'} - \lambda_{r,1'} \times P_{1'}) \times [1 - \cos(2 \times \beta_{1'})] \quad (\text{B.23})$$

$$M_{\text{lat},1'} = \lambda_{r,1'} \times P_{1'} + 0,5 \times (\lambda_{t,1'} \times P_{1'} - \lambda_{r,1'} \times P_{1'}) \times [1 + \cos(2 \times \beta_{1'})] \quad (\text{B.24})$$

$$P0: M_{\text{long, neigh}} = \Sigma M_{\text{long},j} \quad (\text{B.25})$$

$$M_{\text{lat, neigh}} = \Sigma M_{\text{lat},j} \quad (\text{B.26})$$

For the above formulae:

$\lambda_{r,1}$ and $\lambda_{t,1}$ are defined using $x_1 = a$;

$\lambda_{r,0'}$ and $\lambda_{t,0'}$ are defined using $x_{0'} = s$;

$\lambda_{r,1'}$ and $\lambda_{t,1'}$ are defined using $x_{1'} = \frac{a}{\cos \beta_{1'}}$ and $\beta_{1'} = \tan^{-1}(\frac{s}{a})$.

B.1.5.3 Longitudinal bending tensile stress σ_{long} [N/mm²] and lateral bending tensile stress σ_{lat} [N/mm²] due to rail seat loads

System variant I: single layer (thickness h_I) on substructure

Longitudinal and lateral bending tensile stress at bottom of slab:

$$\sigma_{\text{long I}} = \frac{6 \times M_{\text{long I}}}{h_I^2} \text{ and } \sigma_{\text{lat I}} = \frac{6 \times M_{\text{lat I}}}{h_I^2} \quad (\text{B.27})$$

System variant II: unbonded multiple layers (material properties: E_i, h_i) with halfspace equivalent thickness h_{II} on substructure

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$M_{\text{long I}} = M_{\text{long II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.28})$$

$$M_{\text{lat I}} = M_{\text{lat II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.29})$$

$$M_{\text{long } 2} = M_{\text{long II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.30})$$

$$M_{\text{lat } 2} = M_{\text{lat II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.31})$$

$$\sigma_{\text{long } 1} = 6 \times \frac{M_{\text{long } 1}}{h_1^2}, \sigma_{\text{long } 2} = 6 \times \frac{M_{\text{long } 2}}{h_2^2} [\text{N} / \text{mm}^2] \quad (\text{B.32})$$

$$\sigma_{\text{lat } 1} = 6 \times \frac{M_{\text{lat } 1}}{h_1^2}, \sigma_{\text{lat } 2} = 6 \times \frac{M_{\text{lat } 2}}{h_2^2} [\text{N} / \text{mm}^2] \quad (\text{B.33})$$

where

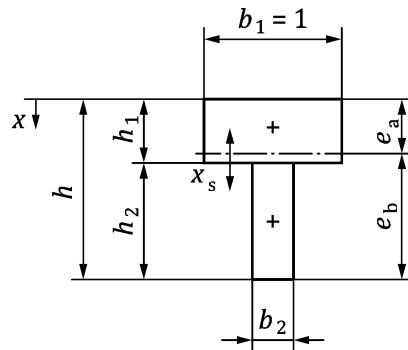
$M_{\text{long II}}$ and $M_{\text{lat II}}$ are the bending moments [Nmm] calculated using the Westergaard approach;

h_1 and h_2 are the thicknesses [mm] of layers 1 and 2;

E_1 and E_2 are the Young's moduli [N/mm²] of layers 1 and 2.

System variant III: fully bonded multiple layers (material properties: E_i , h_i) with half-space equivalent thickness h_{III} on substructure

Model: T-beam (with equivalent stiffness $E=E_1$)



Key

- h thickness of both bonded layers
- h_1 thickness of first layer
- h_2 thickness of second layer
- b_1 reference width of first layer
- b_2 equivalent width of second layer
- e_a distance between surface of first layer and neutral axis of the model T-beam
- e_b distance between neutral axis of the model T-beam and the bottom of the second layer
- x_s distance between any point of interest and the neutral axis of the model T-beam

Figure B.11 — T-beam model for equivalent layer stiffness

NOTE The formulae are applicable in case the neutral axis of the composite section is inside the top layer.

$$b_2 = \frac{E_2}{E_1} \quad (B.34)$$

$$e_a = \frac{\sum (A_i \times x_i)}{\sum A_i} [\text{mm}], \text{ where: } A_1 = 1 \times h_1 [\text{mm}^2] \text{ and } A_2 = b_2 \times h_2 [\text{mm}^2] \quad (B.35)$$

$$e_b = h - e_a [\text{mm}], \text{ where } h = h_1 + h_2 [\text{mm}] \quad (B.36)$$

The moment of inertia of a T-beam [mm⁴ per mm]:

$$I = \sum (I_i + A_i \times x_{s,i}^2) [\text{mm}^4] \text{ where } I_i = \frac{b_i \times h_i^3}{12} [\text{mm}^4] \quad (B.37)$$

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$\sigma_{\text{long1,top}} = \frac{M_{\text{long III}}}{I} \times e_a [\text{N} / \text{mm}^2] \quad (B.38)$$

$$\sigma_{\text{long1,bottom}} = \frac{M_{\text{long III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.39)$$

$$\sigma_{\text{long2,top}} = b_2 \times \frac{M_{\text{long III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.40)$$

$$\sigma_{\text{long2,bottom}} = b_2 \times \frac{M_{\text{long III}}}{I} \times e_b [\text{N} / \text{mm}^2] \quad (B.41)$$

$$\sigma_{\text{lat1,top}} = \frac{M_{\text{lat III}}}{I} \times e_a [\text{N} / \text{mm}^2] \quad (B.42)$$

$$\sigma_{\text{lat1,bottom}} = \frac{M_{\text{lat III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.43)$$

$$\sigma_{\text{lat2,top}} = b_2 \times \frac{M_{\text{lat III}}}{I} \times (h_1 - e_a) [\text{N} / \text{mm}^2] \quad (B.44)$$

$$\sigma_{\text{lat2,bottom}} = b_2 \times \frac{M_{\text{lat III}}}{I} \times e_b [\text{N} / \text{mm}^2] \quad (B.45)$$

where

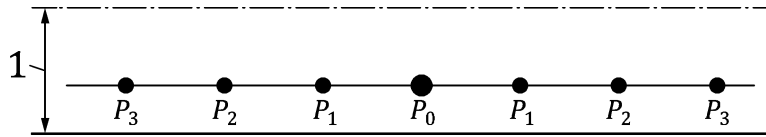
$M_{\text{long III}}$ and $M_{\text{lat III}}$ are the bending moments [Nmm] calculated using the Westergaard approach;

h_1 and h_2 are the thicknesses [mm] of layers 1 and 2;

E_1 and E_2 are the Young's moduli [N/mm²] of layers 1 and 2.

B.1.6 Beam on Winkler foundation (Zimmermann): Longitudinal bending moment and tensile stress due to rail seat loads

B.1.6.1 General



Key

1 1/2 width

Figure B.12 — Beam model, rail seat configuration

B.1.6.2 Longitudinal bending moment $M_{\text{long I,II,III}}$ [Nmm] due to rail seat loads

B.1.6.2.1 General

Longitudinal bending moment $M_{\text{long I,II,III}}$ [Nmm] activated by the traffic load at rail seat location 0 may be computed from the bending moment $M_0 \text{ I,II,III}$ [Nmm], due to rail seat load P_0 [N], and the longitudinal bending moment $M_{\text{long neigh}}$ [Nmm], due to neighbouring rail seat load P_j [N].

$$M_{\text{long I,II,III}} = M_0 \text{ I,II,III} + M_{\text{long neigh}}$$

B.1.6.2.2 Bending moment $M_0 \text{ I,II,III}$ [Nmm] due to rail seat load P_0 [N]

Using Zimmermann calculation, the bending moment in the infinitely long concrete beam due to load P_0 is given below. Since the system is symmetrical in transverse direction, any half-side of the slab may be used.

$$M_{0 \text{ I,II,III}} = \frac{P_0 \times L_{\text{el}}}{4} \quad (\text{B.46})$$

The maximum rail seat load P_0 [N] is calculated according to Annex A.

The elastic length L_{el} [mm] of the beam is given below:

$$L_{\text{el}} = \sqrt[4]{\frac{4 \times E_1 \times I_B}{b_B \times k}} \quad (\text{B.47})$$

where

E_1 is the Young's modulus [N/mm²] of concrete beam [N/mm²];

k is the bedding modulus [N/mm³].

The moment of inertia of the beam I_B [mm⁴] is given below:

$$I_B = \frac{b_B \times h^3}{12} \quad (\text{B.48})$$

where

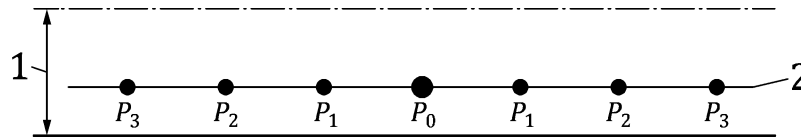
b_B is half of slab width [mm];

h is thickness [mm] of system h_I or h_{II} or h_{III} .

B.1.6.2.3 Longitudinal bending moment $M_{\text{long neigh}}$ [Nmm] due to neighbouring rail seat load P_j [N]

Check on additional longitudinal bending moments in the concrete beam shall be made using the Zimmermann approach. Depending on the elastic length L_{el} , neighbouring rail seat loads increase the bending moment at rail seat location 0.

Using the approach, a beam of width equal to half of the slab width (any half, left or right, may be used) and thickness h_1 (as earlier defined) is assumed to be loaded vertically by the rail seat loads P_j , see Figure B.13.



Key

- 1 1/2 width
- 2 left/right rail

Figure B.13 — Rail seat configuration

Bending moment at 0 due to neighbouring loads P_j at distances x_j from the reference load P_0 :

$$M_{\text{long neigh}} = \frac{L_{el}}{4} \times \sum (P_j \times \mu_j) [\text{Nmm}] \quad (\text{B.49})$$

$$\mu_j = \frac{-\sin \xi_j + \cos \xi_j}{e^{\xi_j}} \quad (\text{B.50})$$

$$\text{with : } \xi_j = x_j / L_{el} \quad (\text{B.51})$$

where

L_{el} is the elastic length [mm] of the beam system;

x_j is the distance [mm] between the rail seat 0 and the rail seat j ;

P_j are the rail seat loads [N] calculated according to Annex A.

Only positive μ_j shall be taken into account.

B.1.6.3 Longitudinal bending tensile stress σ_{long} [N/mm²] due to traffic load

System variant I: single layer (thickness h_I) on substructure

Longitudinal bending tensile stress at bottom of slab:

$$\sigma_{\text{long I}} = \frac{6 \times M_{\text{long I}}}{\frac{B}{2} \times h_1^2} \quad (\text{B.52})$$

where

B is the width [mm] of the slab of the single layer.

System variant II: unbonded multiple layers (material properties: E_i , h_i) with halfspace equivalent thickness h_{II} on substructure

Longitudinal bending tensile stresses in layers 1 and 2:

$$M_{\text{long I}} = M_{\text{long II}} \times \frac{E_1 \times h_1^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.53})$$

$$M_{\text{long 2}} = M_{\text{long II}} \times \frac{E_2 \times h_2^3}{E_1 \times h_1^3 + E_2 \times h_2^3} [\text{Nmm}] \quad (\text{B.54})$$

$$\sigma_{\text{long I}} = 6 \times \frac{M_{\text{long I}}}{\frac{B_1}{2} \times h_1^2}, \sigma_{\text{long 2}} = 6 \times \frac{M_{\text{long 2}}}{\frac{B_2}{2} \times h_2^2} [\text{N} / \text{mm}^2] \quad (\text{B.55})$$

where

$M_{\text{long II}}$ is the bending moment [Nmm] calculated using the Zimmermann approach;

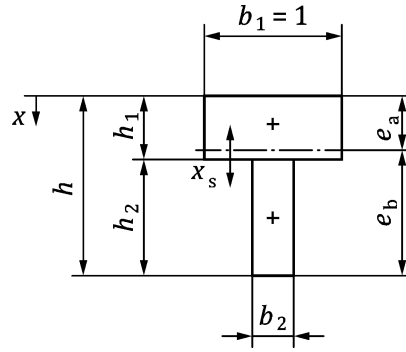
h_1 and h_2 are the thicknesses [mm] of layers 1 and 2;

E_1 and E_2 are the Young's moduli [N/mm^2] of layers 1 and 2;

B_1 and B_2 are the width [mm] of the slabs of layers 1 and 2.

System variant III: fully bonded multiple layers (material properties: E_i , h_i) with half-space equivalent thickness h_{III} on substructure

Longitudinal and lateral bending tensile stresses in layers 1 and 2.



Key

- h thickness of both bonded layers
- h_1 thickness of first layer
- h_2 thickness of second layer
- b_1 reference width of first layer
- b_2 equivalent width of second layer
- e_a distance between surface of first layer and neutral axis of the model T-beam
- e_b neutral axis of the model T-beam and the bottom of the second layer
- x_s distance between any point of interest and the neutral axis of the model T-beam

Figure B.14 — Model T-beam (with equivalent stiffness $E = E_1$)

$$b_2 = \frac{E_2}{E_1} \quad (\text{B.56})$$

$$e_a = \frac{\sum (A_i \times x_i)}{\sum A_i} \text{ mm, where: } A_1 = b_1 \times h_1 \text{ [mm}^2\text{] and } A_2 = b_2 \times h_2 \text{ [mm}^2\text{]} \quad (\text{B.57})$$

$$e_b = h - e_a \text{ [mm], where } h = h_1 + h_2 \text{ [mm]} \quad (\text{B.58})$$

The moment of inertia of a T-beam [mm⁴ per mm]:

$$I = \sum \left(I_i + A_i \times x_{s,i}^2 \right) \text{ [mm}^4\text{]}, \text{ where } I_i = \frac{b_i \times h_i^3}{12} \text{ [mm}^4\text{]} \quad (\text{B.59})$$

Longitudinal and lateral bending tensile stresses in layers 1 and 2:

$$\sigma_{\text{long 1,top}} = \frac{M_{\text{long III}}}{\frac{B_1}{2} \times I} \times (e_a) \text{ [N / mm}^2\text{]}, \sigma_{\text{long 1,bottom}} = \frac{M_{\text{long III}}}{\frac{B_1}{2} \times I} \times (h_1 - e_a) \text{ [N / mm}^2\text{]} \quad (\text{B.60})$$

$$\sigma_{\text{long 2,top}} = b_2 \times \frac{M_{\text{long III}}}{\frac{B_2}{2} \times I} \times (h_1 - e_a) \text{ [N / mm}^2\text{]}; \sigma_{\text{long 2,bottom}} = b_2 \times \frac{M_{\text{long III}}}{\frac{B_2}{2} \times I} \times e_b \text{ [N / mm}^2\text{]} \quad (\text{B.61})$$

where

- $M_{\text{long III}}$ are the bending moment [Nmm] calculated by the Zimmermann approach;
 h_1 and h_2 are the thicknesses [mm] of layers 1 and 2;
 E_1 and E_2 are the Young's moduli [N/mm²] of layers 1 and 2;
 B_1 and B_2 are the width [mm] of the slabs of layers 1 and 2.

B.1.7 Critical longitudinal bending tensile stress

The critical longitudinal bending tensile stress activated at the position of rail seat load P_0 shall be determined. The critical longitudinal bending tensile stress will be the greatest of:

The longitudinal bending tensile stress σ_{long} [N/mm²] calculated using the longitudinal bending moment $M_{\text{long I}}$ or $M_{\text{long II}}$ or $M_{\text{long III}}$ at slab interior (Westergaard approach), and

the longitudinal bending tensile stress σ_{long} [N/mm²] calculated using the longitudinal bending moment $M_{\text{long I}}$ or $M_{\text{long II}}$ or $M_{\text{long III}}$ in half width of the pavement (Zimmermann approach).

B.1.8 Critical lateral bending tensile stress

The critical lateral bending tensile stress activated at the position of rail seat load P_0 is the lateral bending tensile stress σ_{lat} [N/mm²] calculated using the bending moment $M_{\text{lat I}}$ or $M_{\text{lat II}}$ or $M_{\text{lat III}}$ at slab interior by Westergaard approach.

B.2 Stresses in concrete slab/pavement due to thermal impact

B.2.1 General

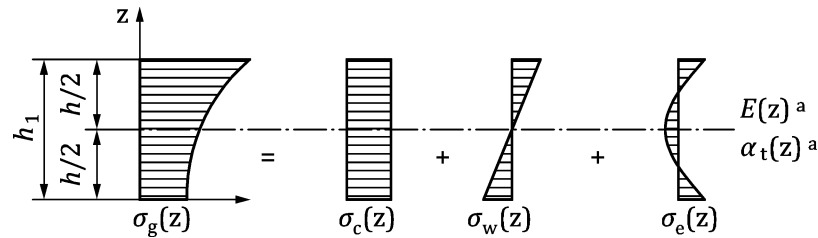
Temperature changes in the subsystem concrete slab/pavement cause deformation and stresses. With respect to potential deformations, the stresses (tensile stresses are decisive) acting along the thickness h should be separated into constant σ_c , linear σ_w and residual σ_e stresses. More specifically:

Constant stresses σ_c : Cooling (ΔT) causes changes of slab-length and slab-width (the longest one is decisive) and/or constant stresses σ_c , due to friction at the bottom interface, and/or longitudinal restraint by longitudinal reinforcement (CRCP). In this case, relevant for the design are the characteristic tensile stresses activated by cooling, during winter time, see B.2.2.

Linear stresses σ_w : Heating causes bending of the slab and remaining warping stresses σ_w , due to linear temperature gradient Δt dependent on the slab dimensions and the support conditions. In this case, relevant for the design are the characteristic bending tensile stresses activated by heating, during summer time, see B.2.3.

Residual stresses σ_e : Residual stresses are not converted into deformations. This stress partition at bottom of the slab is typically in compression and not relevant for design.

The highest stress level calculated by B.2.2 and B.2.3 shall be used as constant stress level to determine the stress limit for traffic load.



NOTE $a = \text{constant}$

Figure B.15 — Stresses in concrete slab/pavement due to thermal impact

B.2.2 Constant stresses σ_c due to temperature changes ΔT acting in concrete slabs or pavements

B.2.2.1 Jointed Plain Concrete Pavements (JPCP)

Constant stresses σ_c due to temperature changes ΔT are not relevant in case slab length / joint spacing is not exceeding 5 m.

B.2.2.2 Continuously Reinforced Concrete Pavements (CRCP) or Jointed Reinforced Concrete Pavements (JRCP)

For reducing the crack width in the concrete slab/pavement continuously reinforced concrete pavements with controlled (JRCP) or random cracking (CRCP) shall be used. A percentage of longitudinal reinforcement equivalent to 0,8 % to 0,9 % and diameter \varnothing 18 mm to 20 mm is recommended for CRCP in order to achieve fine cracks (crack width < 0,5 mm and crack spacing < 5 m) and to avoid merging cracks. JRCP is typically using smaller amount of reinforcement e.g. 0,4 % to 0,5 %.