9 Fatigue limit state (LS4)

9.1 Design values of actions

(1) The design values of the actions for each load case should be taken as the varying parts of the total action representing the anticipated action spectrum throughout the design life of the structure.

(2) The relevant action spectra should be obtained from EN 1991 in accordance with the definitions given in the appropriate application parts of EN 1993.

9.2 Stress design

9.2.1 General

(1) The fatigue assessment presented in EN 1993-1-9 should be used, except as provided here.

(2)P The partial factor for resistance to fatigue γ_{Mf} shall be taken from the relevant application standard.

> **NOTE:** The value of the partial factor γ_{Mf} may be defined in the National Annex. Where no application standard exists for the form of construction involved, or the application standard does not define the relevant values of γ_{Mf}, the value of γ_{Mf} should be taken from EN 1993-1-9. It is recommended that the value of γ_{Mf} should not be taken as smaller than $\gamma_{Mf} = 1{,}1$.

9.2.2 Design values of stress range

(1) Stresses should be determined by a linear elastic analysis of the structure subject to the design values of the fatigue actions.

(2) In each verification of the limit state, the design value of the fatigue stress should be taken as the larger stress range $\Delta\sigma$ of the values on the two surfaces of the shell, and based on the sum of the primary and the secondary stresses.

(3) Depending upon the fatigue assessment carried out according to EN 1993-1-9, either nominal stress ranges or geometric stress ranges should be evaluated.

(4) Nominal stress ranges may be used if 9.2.3 (2) is adopted.

(5) Geometric stress ranges should be used for construction details that differ from those of 9.2.3 (2).

(6) The geometric stress range takes into account only the overall geometry of the joint, excluding local stresses due to the weld geometry and internal weld effects. It may be determined by use of geometrical stress concentration factors given by expressions.

(7) Stresses used for the fatigue design of construction details with linear geometric orientation should be resolved into components transverse to and parallel to the axis of the detail.

9.2.3 Design values of resistance (fatigue strength)

(1) The design values of resistance obtained from the following may be applied to structural steels in the temperature range up to 150° C.

(2) The fatigue resistance of construction details commonly found in shell structures should be obtained from EN 1993-3-2 in classes and evaluated in terms of the stress range $\Delta\sigma_E$, appropriate to the number of cycles, in which the values are additionally classified according to the quality of the welds.

(3) The fatigue resistance of the detail classes should be obtained from EN 1993-1-9.

9.2.4 Stress range limitation

(1) In every verification of this limit state, the design stress range $\Delta\sigma_E$ should satisfy the condition:

$$\gamma_{Ff}\ \Delta\sigma_E \leq \Delta\sigma_R / \gamma_{Mf} \qquad \text{... (9.1)}$$

where:

γ_{Ff}	is	the partial factor for the fatigue loading
γ_{Mf}	is	the partial factor for the fatigue resistance
$\Delta\sigma_E$	is	the equivalent constant amplitude stress range of the design stress spectrum
$\Delta\sigma_R$	is	the fatigue strength stress range for the relevant detail category and the number of cycles of the stress spectrum

(2) As an alternative to (1), a cumulative damage assessment may be made for a set of m different stress ranges $\Delta\sigma_i$ (i = 1,m) using the Palmgren-Miner rule:

$$D_d \leq 1 \qquad \text{... (9.2)}$$

in which:

$$D_d = \sum_{i=1}^{m} n_i / N_i \qquad \text{... (9.3)}$$

where:

n_i	is	the number of cycles of the stress range $\Delta\sigma_i$
N_i	is	the number of cycles of the stress range $\gamma_{Ff}\ \gamma_{Mf}\ \Delta\sigma_i$ to cause failure for the relevant detail category

(3) In the case of combination of normal and shear stress ranges the combined effects should be considered in accordance with EN 1993-1-9.

9.3 Design by global numerical LA or GNA analysis

(1) The fatigue design on the basis of an elastic analysis (LA or GNA analysis) should follow the provisions given in 9.2 for stress design. However, the stress ranges due to the fatigue loading should be determined by means of a shell bending analysis, including the geometric discontinuities of joints in constructional details.

(2) If a three dimensional finite element analysis is used, the notch effects due to the local weld geometry should be eliminated.

ANNEX A (normative)
Membrane theory stresses in shells

A.1 General

A.1.1 Action effects and resistances

The action effects or resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the action effect or resistance when characteristic values of the actions, geometric parameters and material properties are adopted.

A.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

F_x	axial load applied to the cylinder
F_z	axial load applied to a cone
M	global bending moment applied to the complete cylinder (not to be confused with the moment per unit width in the shell wall m)
M_t	global torque applied to the complete cylinder
V	global transverse shear applied to the complete cylinder

Roman lower case letters

g	unit weight of the material of the shell
p_n	distributed normal pressure
p_x	distributed axial traction on cylinder wall

Greek lower case letters

ϕ	meridional slope angle
σ_x	axial or meridional membrane stress ($= n_x/t$)
σ_θ	circumferential membrane stress ($= n_\theta/t$)
τ	membrane shear stress ($= n_{x\theta}/t$)

A.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 2.3 and 5.2.2.

(2) For these expressions to be strictly valid, the boundary conditions for cylinders should be taken as radially free at both ends, axially supported at one end, and rotationally free at both ends.

(3) For these expressions to be strictly valid for cones, the applied loading should match a membrane stress state in the shell and the boundary conditions should be taken as free to displace normal to the shell at both ends and meridionally supported at one end.

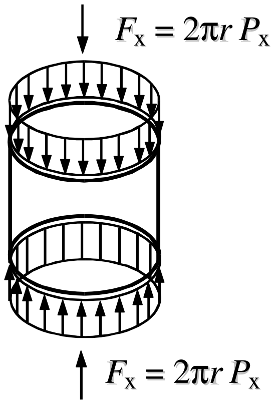
(4) For truncated cones, the boundary conditions should be understood to include components of loading transverse to the shell wall, so that the combined stress resultant introduced into the shell is solely in the direction of the shell meridian.

A.1.4 Sign convention

(1) The sign convention for stresses σ should be taken everywhere as tension positive, though some of the figures illustrate cases in which the external load is applied in the opposite sense.

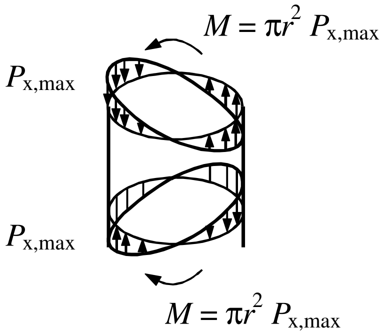
A.2 Unstiffened cylindrical shells

A.2.1 Uniform axial load



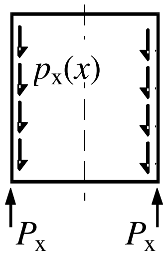
$$\sigma_x = -\frac{F_x}{2\pi r t}$$

A.2.2 Axial load from global bending



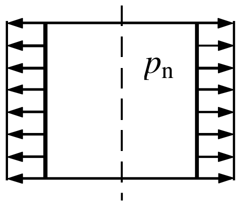
$$\sigma_x = \pm\frac{M}{\pi r^2 t}$$

A.2.3 Friction load



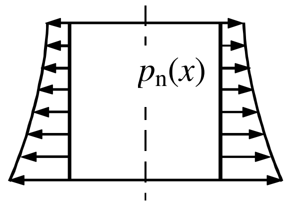
$$\sigma_x = -\frac{1}{t}\int_0^l p_x \cdot dx$$

A.2.4 Uniform internal pressure



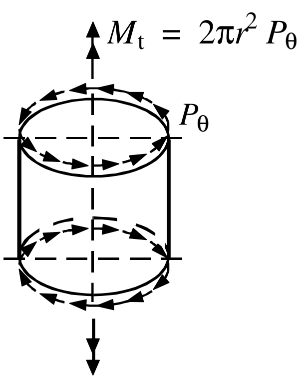
$$\sigma_\theta = p_n \cdot \frac{r}{t}$$

A.2.5 Variable internal pressure



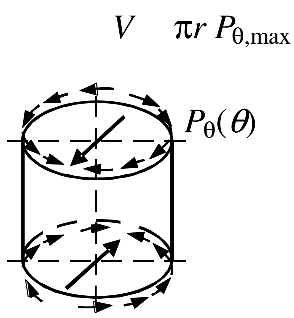
$$\sigma_\theta(x) = p_n(x) \cdot \frac{r}{t}$$

A.2.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

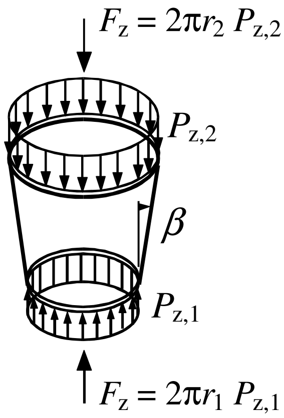
A.2.7 Sinusoidal shear from transverse force



$$\tau_{max} = \pm\frac{V}{\pi r t}$$

A.3 Unstiffened conical shells

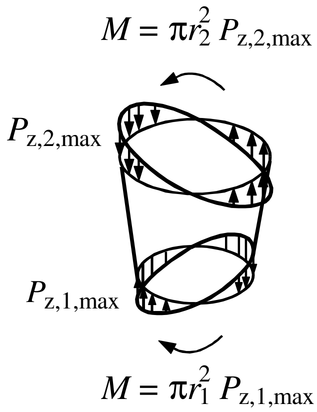
A.3.1 Uniform axial load



$$\sigma_x = -\frac{F_z}{2\pi r t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

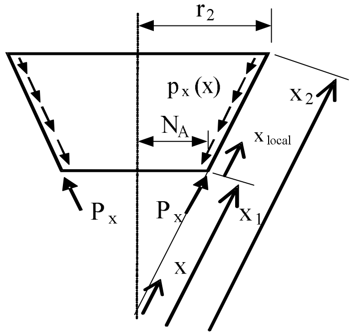
A.3.2 Axial load from global bending



$$\sigma_{x,max} = \pm\frac{M}{\pi r^2 t \cdot \cos\beta}$$

$$\sigma_\theta = 0$$

A.3.3 Friction load

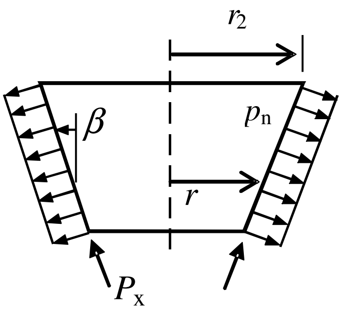


$$x_1 = \frac{r_1}{\sin\beta} \qquad x_2 = \frac{r_2}{\sin\beta}$$

$$\sigma_{x1} = -\frac{1}{x_1 t}\int_{x_1}^{x_2} p_x x \cdot dx$$

$$\sigma_\theta = 0$$

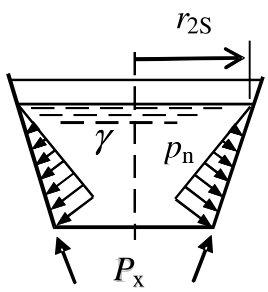
A.3.4 Uniform internal Pressure



$$\sigma_x = -p_n \frac{r}{2t \cdot \cos\beta}\left[\left(\frac{r_2}{r}\right)^2\right]$$

$$\sigma_\theta = p_n \frac{r}{t \cdot \cos\beta}$$

A.3.5 Linearly varying internal pressure

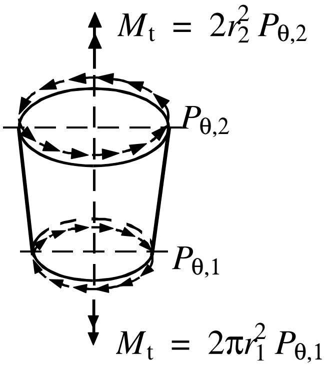


r_{2S} is the radius at the fluid surface

$$\sigma_x = -\frac{\gamma r}{t \cdot \sin\beta}\left\{\frac{r_{2s}}{6}\left[\left(\frac{r_{2s}}{r}\right)^2 - 3\right] + \frac{r}{3}\right\}$$

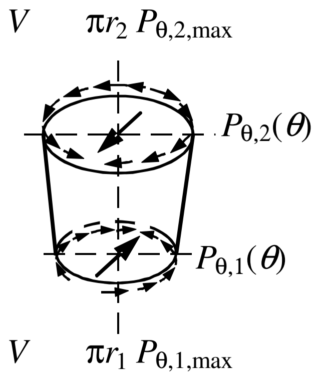
$$\sigma_\theta = +\frac{\gamma r}{t \cdot \sin\beta}(r_{2s} - r)$$

A.3.6 Uniform shear from torsion



$$\tau = \frac{M_t}{2\pi r^2 t}$$

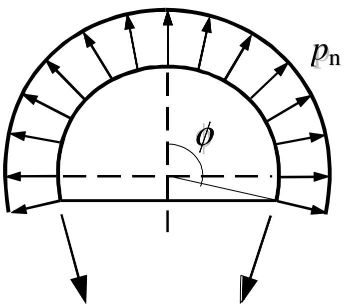
A.3.7 Sinusoidal shear from transverse force



$$\tau_{max} = \pm \frac{V}{\pi r t}$$

A.4 Unstiffened spherical shells

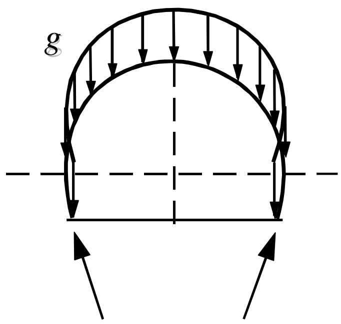
A.4.1 Uniform internal pressure



$$\sigma_x = \frac{p_n r}{2t}$$

$$\sigma_\theta = \frac{p_n r}{2t}$$

A.4.2 Uniform self-weight load



$$\sigma_x = -\frac{gr}{t}\left(\frac{1}{1+\cos\phi}\right)$$

$$\sigma_\theta = -\frac{gr}{t}\left(\cos\phi - \frac{1}{1+\cos\phi}\right)$$

ANNEX B (normative)

A1 Additional expressions for plastic reference resistances A1

B.1 General

B.1.1 Resistances

The resistances calculated using the expressions in this annex may be assumed to provide characteristic values of the resistance when characteristic values of the geometric parameters and material properties are adopted.

B.1.2 Notation

The notation used in this annex for the geometrical dimensions, stresses and loads follows 1.4. In addition, the following notation is used.

Roman upper case letters

A_r cross-sectional area of a ring
P_R characteristic value of small deflection theory plastic mechanism resistance

Roman lower case letters

b thickness of a ring
ℓ effective length of shell which acts with a ring
r radius of the cylinder
s_e dimensionless von Mises equivalent stress parameter
s_m dimensionless combined stress parameter
s_x dimensionless axial stress parameter
s_θ dimensionless circumferential stress parameter

Subscripts

r relating to a ring
R resistance

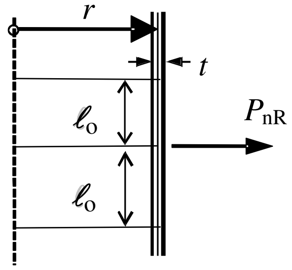
B.1.3 Boundary conditions

(1) The boundary condition notations should be taken as detailed in 5.2.2.

(2) The term “clamped” should be taken to refer to BC1r and the term “pinned” to refer to BC2f.

B.2 Unstiffened cylindrical shells

B.2.1 Cylinder: Radial line load



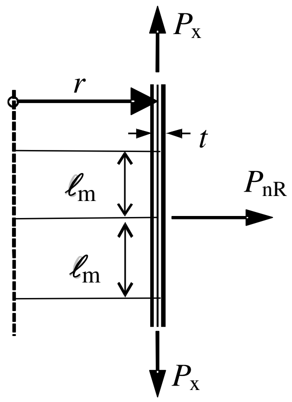
Reference quantities:

$$\ell_o = 0{,}975\sqrt{rt}$$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_0} = f_y \frac{t}{r}$$

B.2.2 Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad \ell_o = 0{,}975\sqrt{rt}$$

Range of applicability:

$$-1 \leq s_x \leq +1$$

Dependent parameters:

If $P_n > 0$ (outward) then: $A = +s_x - 1{,}50$

If $P_n < 0$ (inward) then: $A = -s_x - 1{,}50$

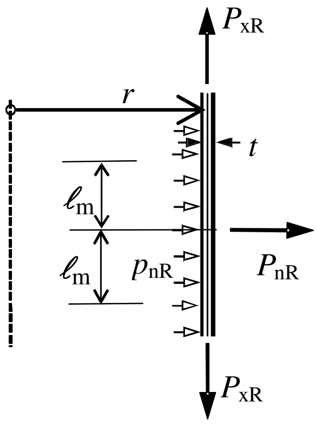
$$s_m = A + \sqrt{A^2 + 4(1 - s_x^2)}$$

If $s_x \neq 0$ then: $\ell_m = s_m \ell_o$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$\frac{P_{nR}}{2\ell_m} = f_y \frac{t}{r}$$

B.2.3 Cylinder: Radial line load, constant internal pressure and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad s_\theta = \frac{p_n}{f_y} \cdot \frac{r}{t}$$

$$\ell_o = 0{,}975\sqrt{rt} \qquad s_e = \sqrt{s_\theta^2 + s_x^2 - s_x s_\theta}$$

Range of applicability:

$$-1 \le s_x \le +1 \qquad -1 \le s_\theta \le +1$$

Dependent parameters:

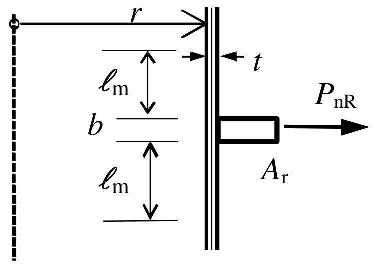
Outward directed ring load $P_n > 0$		Inward directed ring load $P_n < 0$	
Condition	Expressions	Condition	Expressions
$s_e < 1{,}00$ and $s_\theta \le 0{,}975$	$A = +s_x - 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 - s_\theta} \right)$	$s_e < 1{,}00$ and $s_\theta \ge -0{,}975$	$A = -s_x + 2s_\theta - 1{,}50$ $s_m = A + \sqrt{A^2 + 4(1 - s_e^2)}$ $\ell_m = \ell_0 \left(\frac{s_m}{1 + s_\theta} \right)$
$s_e = 1{,}00$ or $s_\theta > 0{,}975$	$\ell_m = 0{,}0$	$s_e = 1{,}00$ or $s_\theta < -0{,}975$	$\ell_m = 0{,}0$

The plastic resistance is given by (P_n and p_n always positive outwards):

$$\frac{P_{nR}}{2\ell_m} + p_n = f_y \frac{t}{r}$$

B.3 Ring stiffened cylindrical shells

B.3.1 Ring stiffened Cylinder: Radial line load

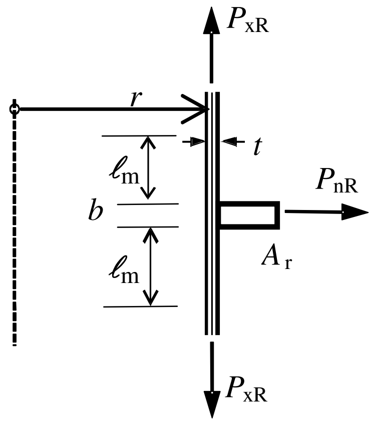


The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$

$$\ell_m = \ell_o = 0{,}975\sqrt{rt}$$

B.3.2 Ring stiffened Cylinder: Radial line load and axial load



Reference quantities:

$$s_x = \frac{P_x}{f_y t} \qquad \ell_o = 0{,}975\sqrt{rt}$$

Range of applicability:

$$-1 \leq s_x \leq +1$$

Dependent parameters:

If $P_n > 0$ then: $A = +s_x - 1{,}50$

If $P_n < 0$ then: $A = -s_x - 1{,}50$

$$s_m = A + \sqrt{A^2 + 4(1 - s_x^2)}$$

If $s_x \neq 0$ then: $\ell_m = s_m \ell_o$

The plastic resistance P_{nR} (force per unit circumference) is given by:

$$P_{nR} = f_y \left(\frac{A_r + (b + 2\ell_m)t}{r} \right)$$