

# Appendix 1

## FILLET DESIGN

### 1. TERMINOLOGY AND SYMBOLS

#### 1.1 General

A description of the terms and symbols used in this appendix is given below. Throughout this appendix it is assumed that the aircraft is taxiing on a horizontal pavement.

#### 1.2 Terms related to the aircraft (see Figure A1-1)

*Centre line through main undercarriage.* Line from the turning centre perpendicular to the aircraft longitudinal axis.

*Datum length ( $d$ ).* Distance between aircraft datum point and centre line through undercarriage.

*Datum point of aircraft ( $S$ ).* Point on longitudinal axis of aircraft which follows the guideline on the ground. The datum point is located vertically beneath the cockpit of the aircraft.

*Main undercarriage centre ( $U$ ).* Point of intersection of longitudinal aircraft axis and centre line through main undercarriage.

*Nose wheel steering angle.* Angle formed by the longitudinal axis of aircraft and the direction of the nose wheel.

*Steering angle ( $\beta$ ).* Angle formed by the tangent to the guideline and the longitudinal axis of aircraft.

*Track of the main undercarriage ( $T$ ).* Distance between the outer main wheels of aircraft including the width of the wheels.

*Turning centre ( $P$ ).* Centre of turn of aircraft at any time.

#### 1.3 Terms related to taxiway and fillet design (see Figure A1-2)

*Deviation of main undercarriage ( $\lambda$ ).* Distance between main undercarriage centre ( $U$ ) and the guideline measured at right angles to the latter.

*Guideline.* Line applied to the pavement by means of markings and/or lights which the aircraft datum point must follow while taxiing.

*Guideline centre ( $O$ ).* Centre of curvature of guideline at point  $S$ .

### 1.4 Glossary of symbols

The following symbols are used when describing the path of the main undercarriage centre and the design of the fillets (see Figures A1-1 and A1-2).

$d$  = aircraft datum length

$M$  = minimum clearance distance between outer wheels of main undercarriage leg and edge of pavement

$O$  = centre of curvature of guideline at point  $S$

$P$  = turning centre

$r$  = radius of fillet arc

$R$  = radius of curvature of guideline at point  $S$

$S$  = datum point of aircraft

$T$  = track of the main undercarriage

$U$  = main undercarriage centre

$\alpha$  = angle between the radial line  $OU$  and the tangent to the path of the main undercarriage centre at  $U$

$\beta$  = steering angle

$\lambda$  = main undercarriage deviation

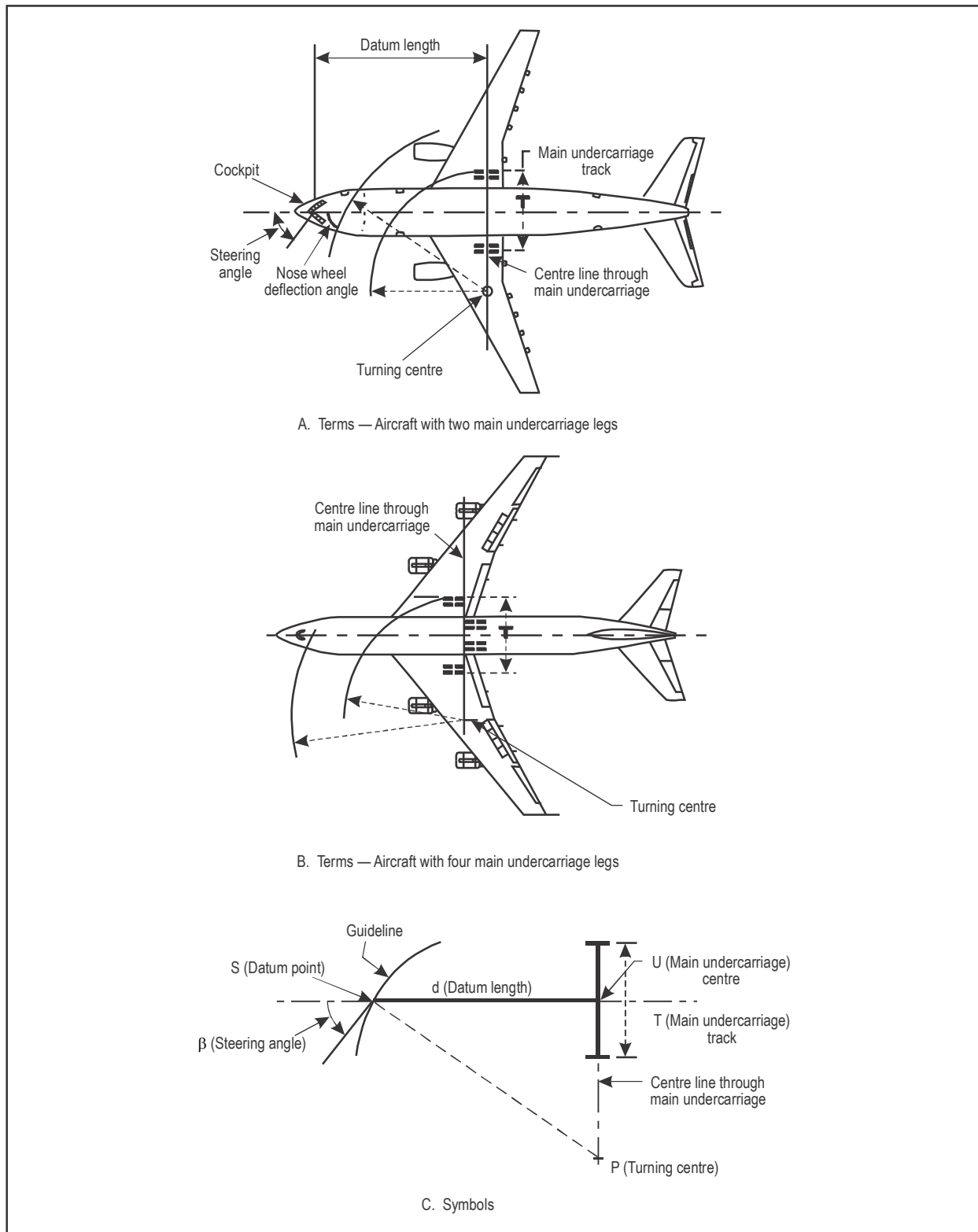
$\rho$  and  $\theta$  = polar coordinates of a point [ $(S)$  or  $(U)$ , as applicable]

## 2. DETERMINATION OF THE PATH FOLLOWED BY THE MAIN UNDERCARRIAGE OF A TAXIING AIRCRAFT

### 2.1 Determination of the path by calculation

#### General

2.1.1 In general, the junction or intersection of taxiways with runways, aprons and other taxiways is achieved by means of an arc of a circle (Figure A1-2B). The calculations below are therefore restricted to the solutions based on this assumption. Nevertheless, the following calculation is more general than the one strictly necessary for the study of fillets. It also applies to movement of an aircraft leaving its parking position on an apron or manoeuvring on a holding bay.



**Figure A1-1. Terms and symbols related to aircraft**

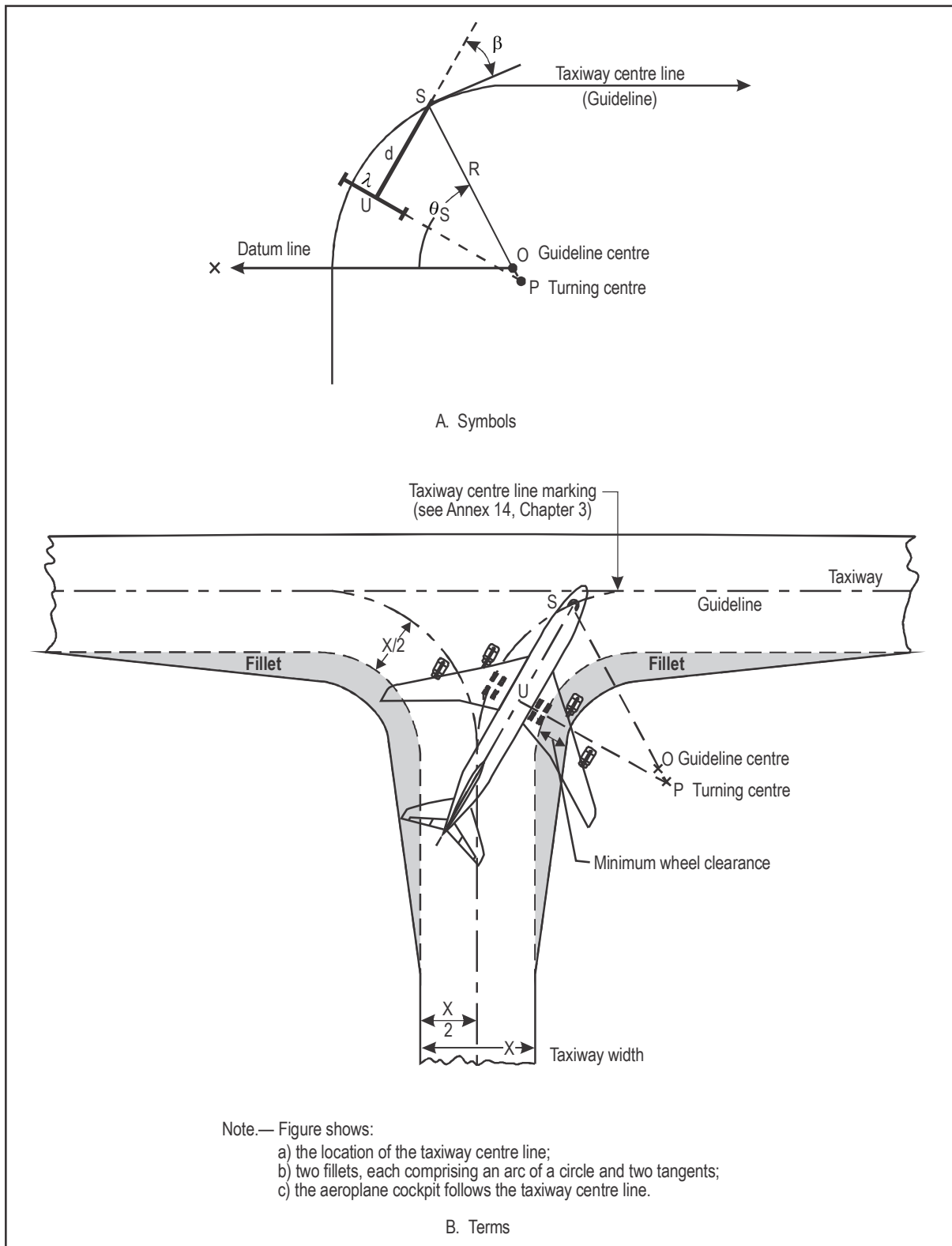


Figure A1-2. Terms and symbols related to taxiway and fillet design

### Datum point (S) follows an arc of a circle

#### Locus of main undercarriage centre (U)

2.1.2 Because of the simplifying assumption above, the datum point of the aircraft (S) follows an arc of a circle with centre O and radius R during the turn. In order to describe the movement of a taxiing aircraft, it is necessary to have a reference coordinate system. Let OX be the datum line,  $\rho$  and  $\theta_U$  be the polar coordinates of U (see Figure A1-3). During movement, the straight line US remains a tangent to the path of the point U at U. This condition produces the differential equation for the locus of U:

$$\tan \alpha = \frac{\rho(d\theta_U)}{(d\rho)} \quad (1)$$

$\rho$  can be expressed as a function of  $d$ ,  $R$  and  $\alpha$  as follows:

$$\rho = d \times \cos \alpha \pm \sqrt{(d^2 \times \cos^2 \alpha - d^2 + R^2)} \quad (2)$$

*Note.— The sign must be positive (+) if  $\alpha > \pi/2$  and negative (–) if  $\alpha < \pi/2$ .*

Separating the variables enables the differential equation (1) to be rewritten as follows:

$$d\theta_U = \frac{d \times \tan \alpha \times \sin \alpha}{\sqrt{[R^2 + d^2 \times (\cos^2 \alpha - 1)]}} \times (d\alpha) \quad (3)$$

Integrating formula (3) produces a biunivocal relationship between  $\theta_U$  and  $\alpha$  under the initial given conditions.

$$\theta_U - \theta_O = \int_{\alpha_O}^{\alpha} \frac{\tan \alpha \times \sin \alpha}{\sqrt{\left[\left(\frac{R^2}{d}\right) + \cos^2 \alpha - 1\right]}} \times (d\alpha) \quad (4)$$

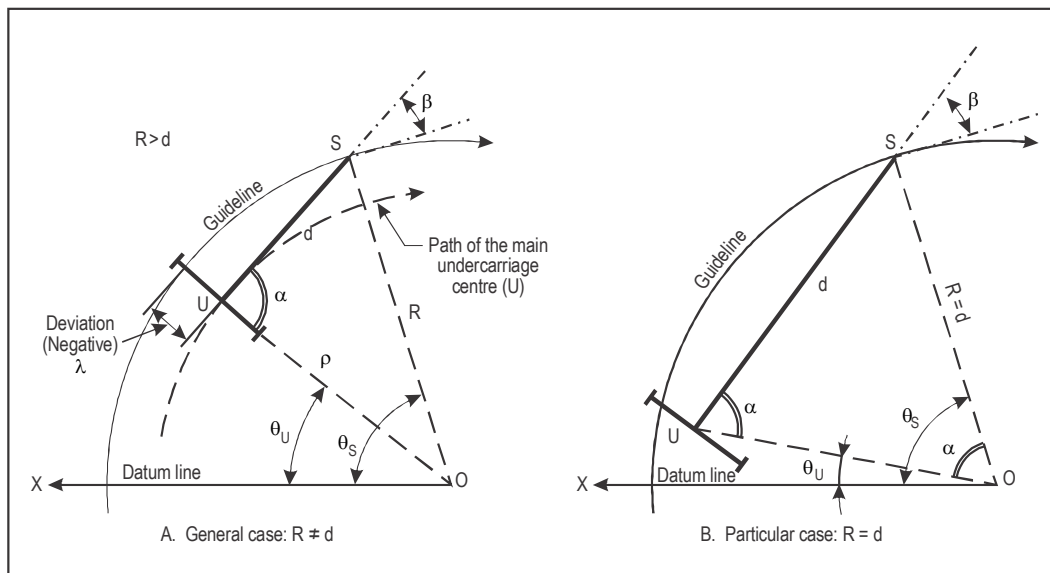


Figure A1-3. Study of the path of the main undercarriage

2.1.3 *Special case:  $R = d$ .* Integration is only easy in the particular case when  $R = d$  (see Figure A1-3B). Indeed, if  $R$ , the radius of curvature of the guideline, is equal to  $d$ , then the datum length of the aircraft would be:

$$\begin{aligned}\theta_U - \theta_S &= \int_{\alpha_0}^{\alpha} \frac{\tan \alpha \times \sin \alpha}{\cos \alpha} \times (d\alpha) \\ &= \int_{\alpha_0}^{\alpha} \tan^2 \alpha \times (d\alpha) = [\tan \alpha - \alpha]_{\alpha_0}^{\alpha}\end{aligned}$$

and by assuming the initial conditions  $\theta_0 = 0$ ,  $\alpha_0 = 0$  and  $\rho_0 = 2d$

$$\theta_U = \tan \alpha - \alpha \quad (5)$$

the angles being expressed in radians. The polar angle of the datum point (S) is then:

$$\theta_s = \tan \alpha \quad (6)$$

The corresponding steering angle is:

$$\beta = 2\alpha - \pi/2 \quad (7)$$

and the deviation of the undercarriage can be calculated by means of the formula:

$$\lambda = d(2 \cos \alpha - 1) \quad (8)$$

The curves for this particular case are plotted on Figure A1-4. The use is explained in 2.2.

2.1.4 *General case:  $R \neq d$ .* If  $R$  is not equal to  $d$ , equation (4) can only be evaluated by solving an elliptical integral. Such an evaluation requires appreciable calculations which cannot be justified for the purpose of fillet design. The alternative method using an approximation described in 2.1.2 equation (4) avoids excessively laborious calculation and still provides a fillet design of adequate accuracy.

2.1.5 Knowledge of the steering angle ( $\beta$ ) at any point of the path of the aircraft datum point (S) easily enables the locus of the main undercarriage centre (U) to be found and hence the path of the undercarriage during the turn to be derived. Let now O be the guideline centre and R its radius. Assuming that the steering angle ( $\beta$ ) remains unchanged, the instantaneous centre of rotation of the aircraft at a given time is P and not O. Consequently, during the short taxi run, the datum point would have departed from the guideline and covered an arc subtending a small angle equal to:

$$\frac{R}{d} \sin \beta \times (d\theta_s) \quad (9)$$

where

$d$  is the datum length of the aircraft;

$R$  and  $\theta_s$  are the polar coordinates of the point S with reference to the datum line OX.

As a first approximation it can be accepted that, when the datum point (S) follows the guideline, the variation in the steering angle is:

$$d\beta = \left(1 - \frac{R}{d} \sin \beta\right) \times (d\theta_s) \quad (10)$$

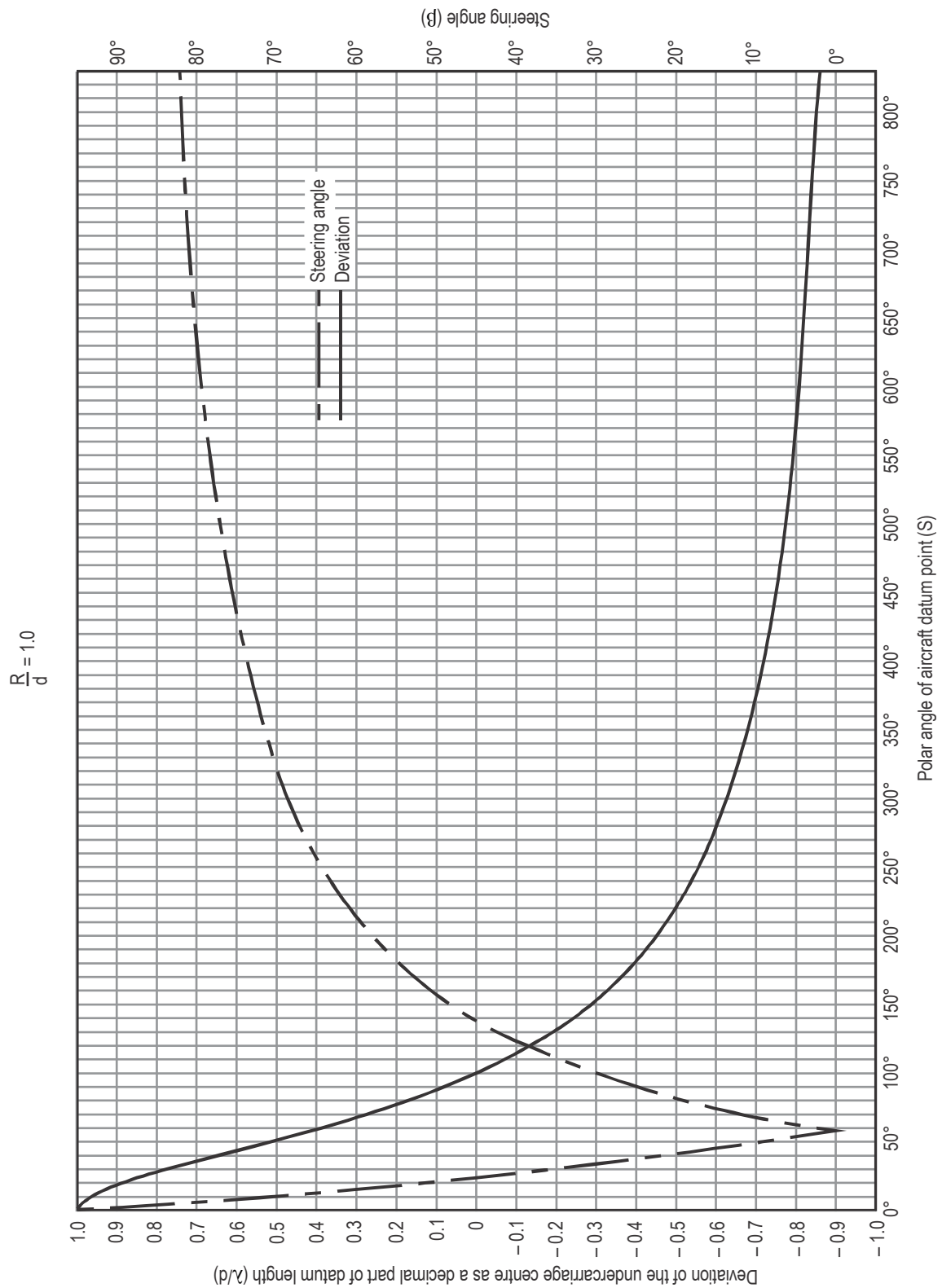


Figure A1-4. Steering angle and deviations of the undercarriage when  $R = d$

This condition produces the following biunivocal relationship between  $\theta_s$  and  $\beta$  under the given initial conditions:

$$\theta_s - \theta_o = \int_{\beta_o}^{\beta} \frac{d}{d - R \sin \beta} \times (d\beta) \quad (11)$$

2.1.6 Integration of this equation prompts the assumption that  $R/d = X$  and consideration of the one case when  $R > d$ ,  $K = \sqrt{X^2 - 1}$ . Solving the equation with respect to  $\beta/2$  and applying the initial conditions  $\theta_o = 0$ ,  $\beta_o = 0$ , it can be found that:

$$\theta_s = \frac{1}{K} \log \frac{1 + (K - S) \tan \frac{\beta}{2}}{1 + (K + X) \tan \frac{\beta}{2}} \quad (12)$$

which, expressed in terms of parameters  $R$  and  $d$ , gives:

$$\theta_s = \frac{d}{\sqrt{R^2 - d^2}} \times \log \frac{d + [\sqrt{R^2 - d^2} - R] \tan \frac{\beta}{2}}{d - [\sqrt{R^2 - d^2} + R] \tan \frac{\beta}{2}} \quad (13)$$

in which  $\theta_s$  is expressed in radians and natural logarithms are used. This allows  $\tan \beta/2$  to be obtained as a function of  $\theta_s$ . Using the above notations it can be found:

$$\tan \frac{\beta}{2} = \frac{1 - e^{K\theta}}{X - K - X \times e^{K\theta} - K \times e^{K\theta}} \quad (14)$$

assuming that  $R > d$ .

#### Deviation of main undercarriage centre ( $\lambda$ )

2.1.7 On an apron, depending on the initial conditions, the deviation of  $U$  can be inside or outside the guideline followed by  $S$  (see Figure A1-5). On a runway or a taxiway when the aircraft datum point ( $S$ ) enters the turn, the initial deviation of the main undercarriage centre is outside the arc of circle and during the turn it gradually tracks in. At any time therefore (see Figure A1-3):

$$\begin{aligned} \frac{\Lambda}{USO} &= \frac{\pi}{2} \pm \beta; \text{ and} \\ (R + \lambda)^2 &= R^2 + d^2 - 2dR \times \cos \left( \frac{\pi}{2} \pm \beta \right) \end{aligned} \quad (15)$$



The solutions of this equation produce the following deviation values:

inside the arc

$$\lambda = \sqrt{(R^2 + d^2 + 2dR \sin \beta)} - R; \text{ and}$$

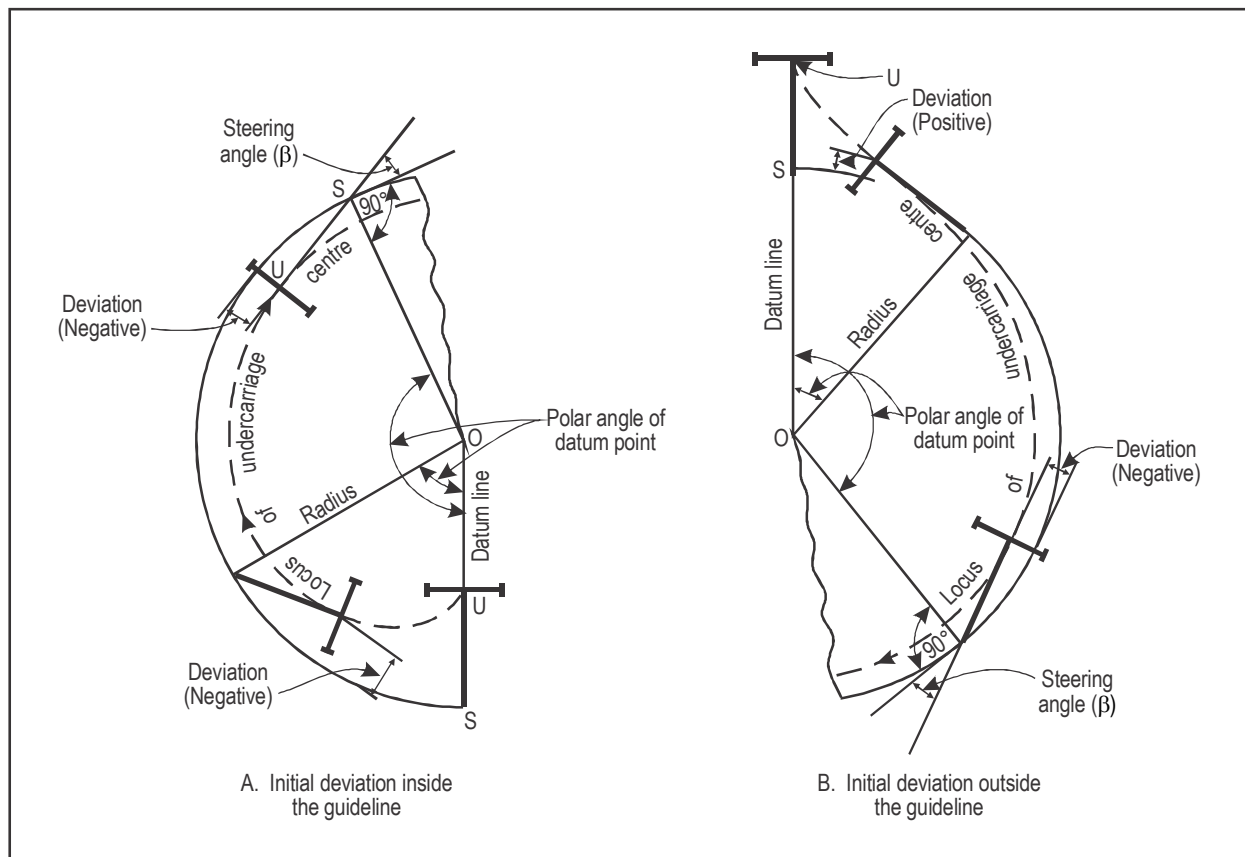
outside the arc

$$\lambda = \sqrt{(R^2 + d^2 + 2dR \sin \beta)} - R; \text{ or}$$

if the deviation value is expressed as a percentage of the aircraft datum length:

$$\frac{\lambda}{d} = \sqrt{(1 + X^2 \pm 2X \sin \beta)} - X \quad (16)$$

where the positive sign must be used in case of deviation outside the arc of circle and the negative sign in case of deviation inside the arc of circle.



**Figure A1-5. Deviation of main undercarriage centre when the datum point follows an arc of circle**

**Datum point (S) follows a straight line***Locus of main undercarriage centre (U)*

2.1.8 Having completed the curve, the datum point (S) follows a straight path along the taxiway centre line. The steering angle is progressively reduced and the main undercarriage centre describes a tractrix (see Figure A1-6). As a result,

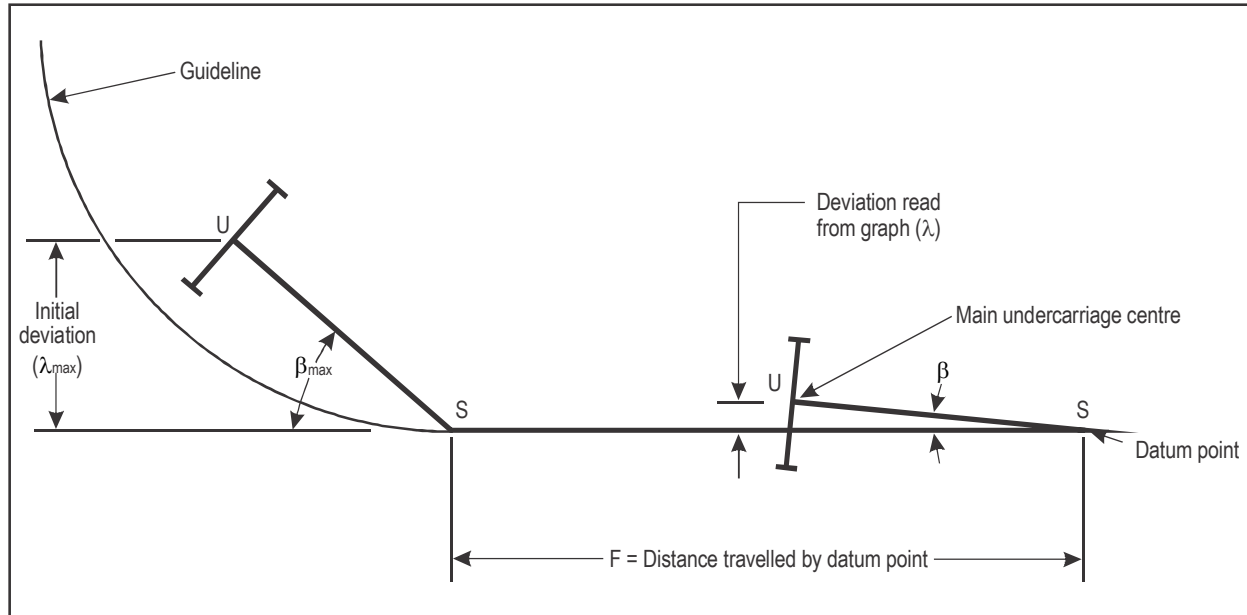
$$\log \tan \frac{\beta}{2} = \log \tan \frac{\beta_{\max}}{2} - \frac{F}{d} \quad (17)$$

enables the steering angle to be calculated when the datum point (S) has travelled through a distance  $F$  along the straight taxiway centre line.

*Deviation of main undercarriage centre ( $\lambda$ )*

2.1.9 When the datum point (S) has covered the distance  $F$  along a straight segment of the guideline (see Figure A1-6) the steering angle ( $\beta$ ) has assumed the value calculated in the first equation of 2.1.3 and the deviation of the main undercarriage centre (U) is given by:

$$\frac{\lambda}{d} = \sin \beta \quad (18)$$



**Figure A1-6. Deviation of the main undercarriage centre when the datum point follows a straight line**